Distributed Signal Decorrelation in WSNs Using the Sparse Matrix Transform (SMT)

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Distributed Anomaly Detection

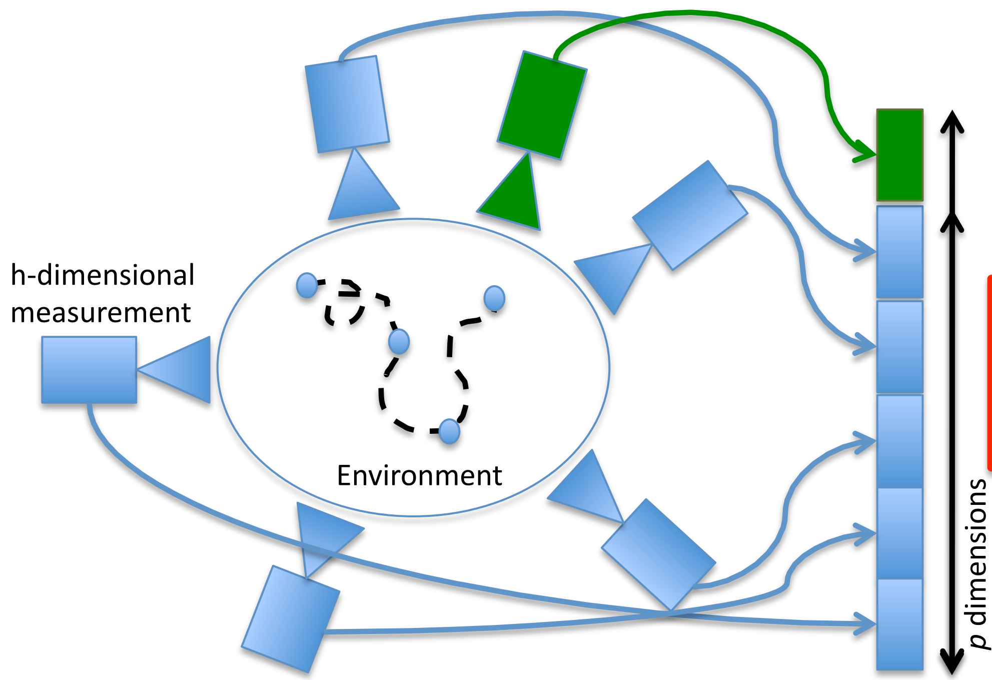
Anomaly detection is

– **Important:** Central to Detection theory
– **Ubiquitous:** Many applications in security-related areas
  - Remote Sensing, Surveillance, Network Intrusion Detection, etc…

Wireless Network of Cameras

- Collectively monitor the environment
- Each outputs image (*vector*) from own viewpoint

Goal: detect anomalies based on ***joint*** measurements from ***all cameras***



Decorrelation requires:

O(p^2) computation/communication

N >> *p* samples to design transform

Big problems

Approach: Sparse Matrix Transform(SMT)

- Allows covariance to be estimated when *n<<p*
  - Imposes sparsity constraint in non-linear manifold
  - Maintains full rank of covariance estimate

✔ Works when *n<<p*

- Results in a fast decorrelating transformation
  - Computation of transform is *O(p)*
  - Generalization of FFT and orthonormal wavelet transform

✔ Fast decorrelation *O(p)*

- **Problem:** Requires lots of communications between sensors

In this paper: The Vector SMT

- Improvement on original SMT
- Suitable for implementation in a network of sensors
  - Distributed *in network* implementation
  - Restrict communication between pairs of sensors

Covariance Estimation Framework

- **Data:** We observe *n* independent *N(0,R)* vectors, each of dimension *p*.

$$Y = [y_1, \cdots, y_n]$$

- **Sample Covariance:**

$$S = \frac{1}{n} Y Y^t$$

- **Model:** Covariance can be represented by

$$R = \mathbf{E}[S] = E \Lambda E^t$$

E – *eigen transform*

Λ – *eigenvalues*

- **Maximum Likelihood (ML) Estimate:**

$$\hat{E} = \arg\min_{E \in \Omega_K} \left\{ \left| diag(E^t S E) \right| \right\}$$

$$\hat{\Lambda} = diag(\hat{E}^t S \hat{E})$$

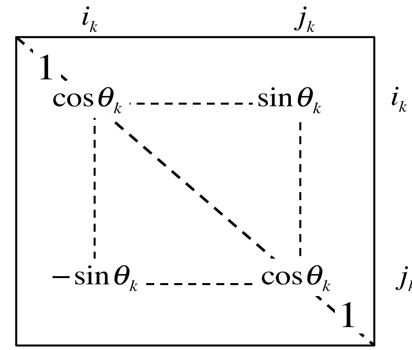
Unconstrained minimization ⟹ PCA of S

Big Idea: Constrain Ω_K ⟹ SMT of order K

The Sparse Matrix Transform (SMT)

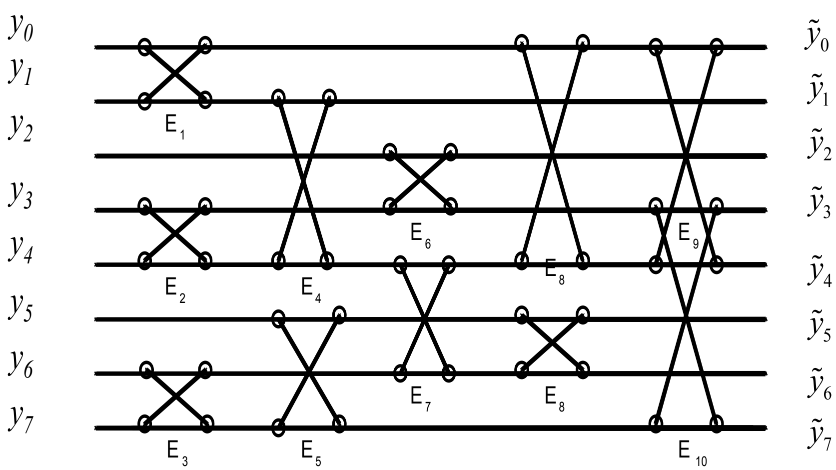
- An SMT is a product of Givens rotations

$$E = \prod_{k=1}^{K} E_k = E_1 \cdots E_K$$, where $E_k =$

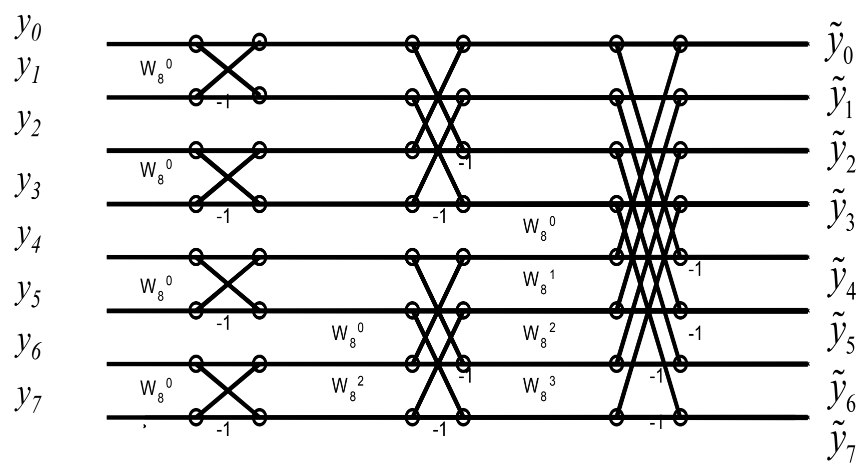


$K = r \cdot p$, where *r* is typically a small constant

- SMT is a generalization of the FFT



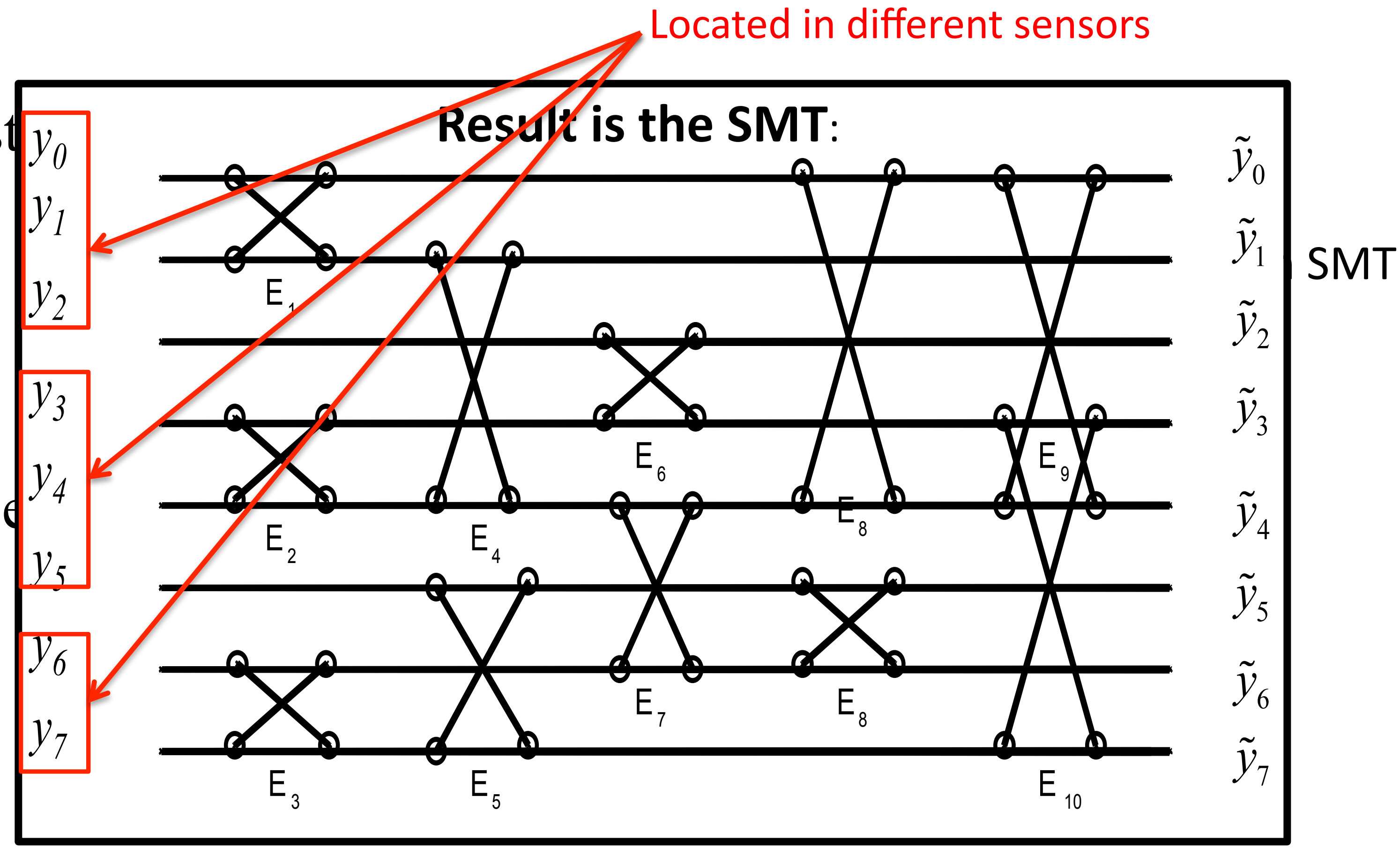
SMT



FFT

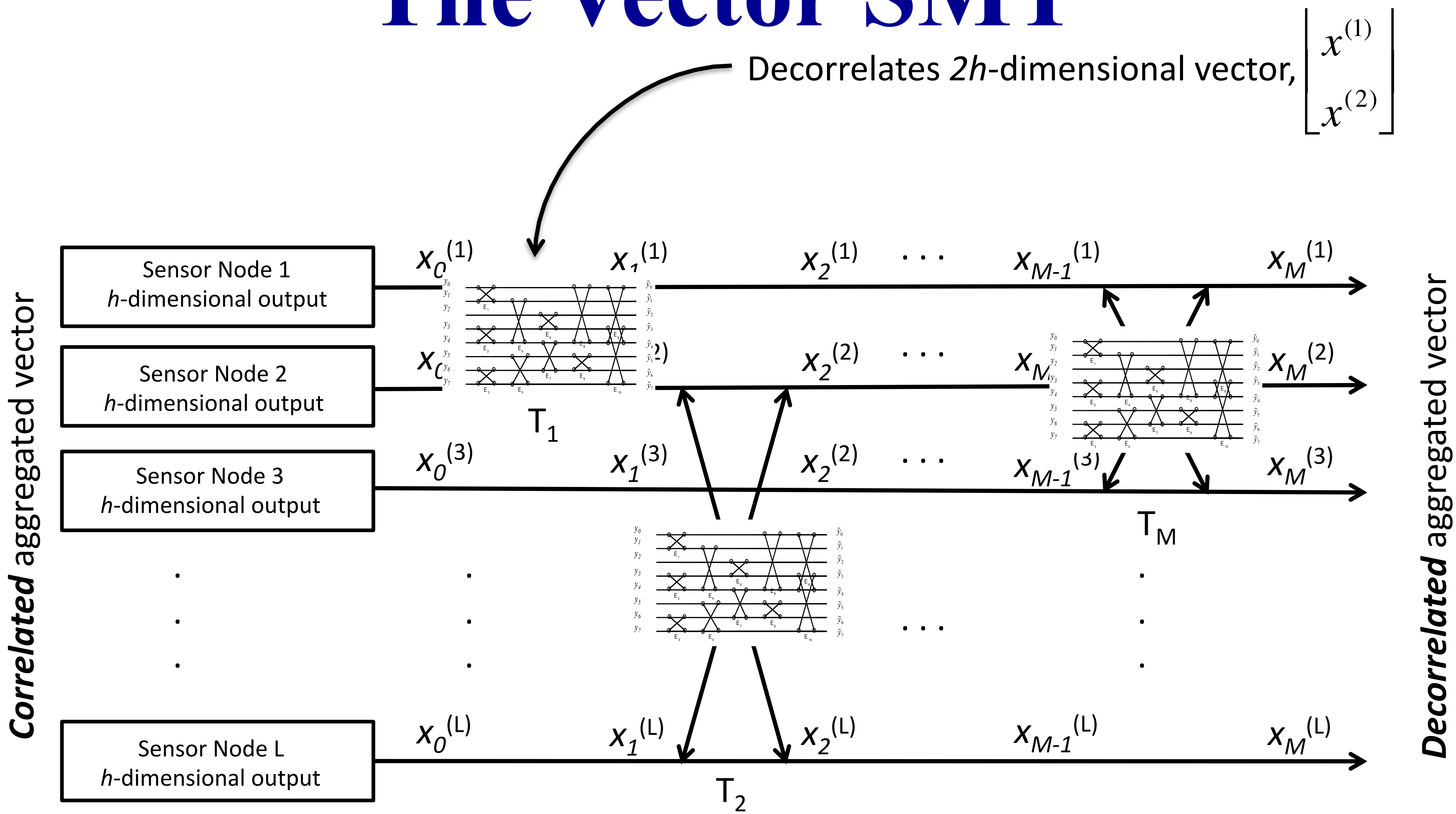
- SMT is also a generalization of the orthonormal (paraunitary) wavelet transform

Design of SMT using Cost Optimization

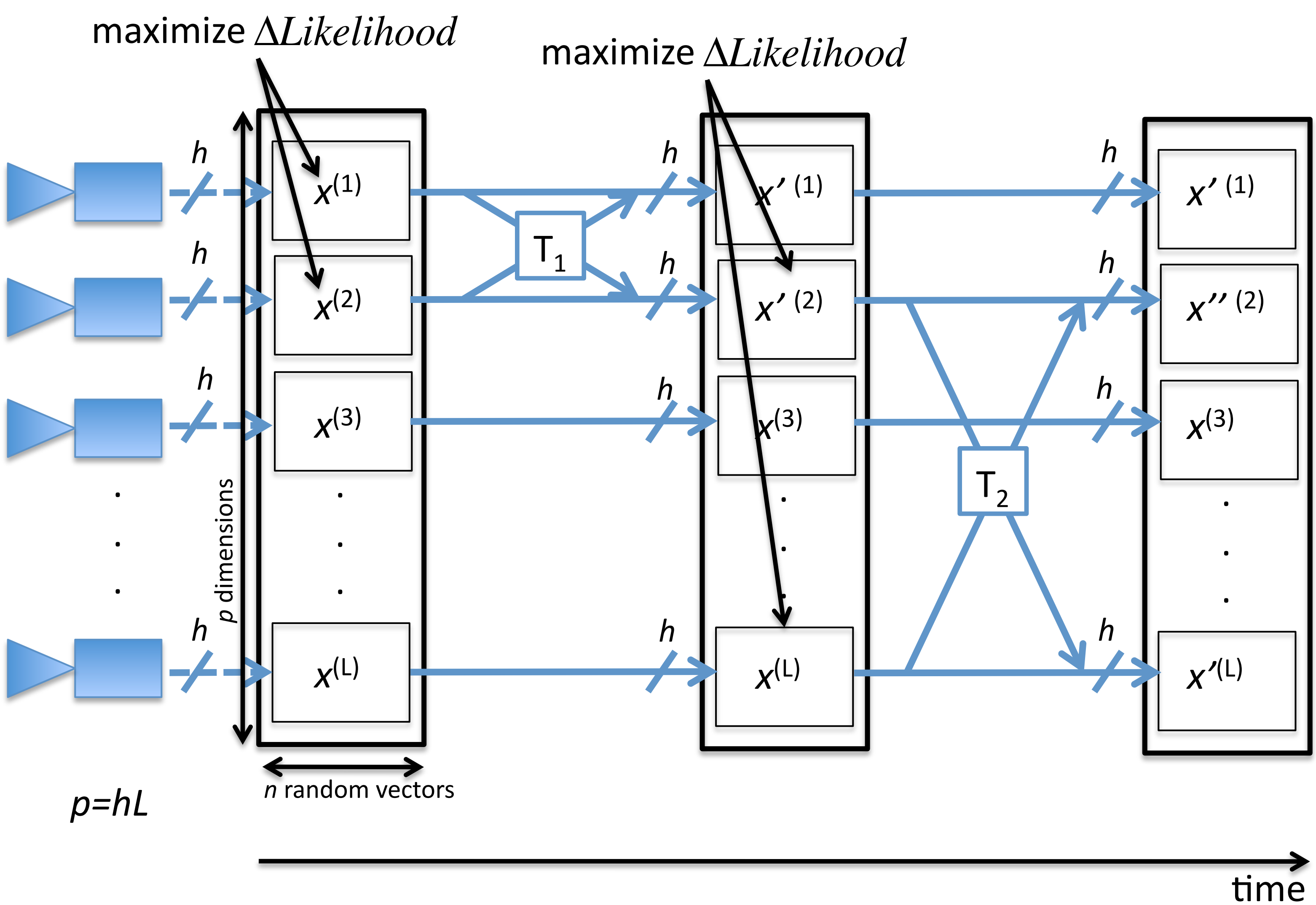


- Cost
- Gree
- Decorrelating transform, $K=rp$ $\Rightarrow$ $O(p)$ computation

The Vector SMT



Vector SMT Design in Data Domain



Anomaly Detection with the Vector SMT

- Vector SMT: models parent distribution of *typical* data
- *Anomaly Detection*: significance test against parent distribution
- Gaussian with covariance R
    - Measure of anomalousness: $x^T R^{-1} x$

Metric for detection accuracy:

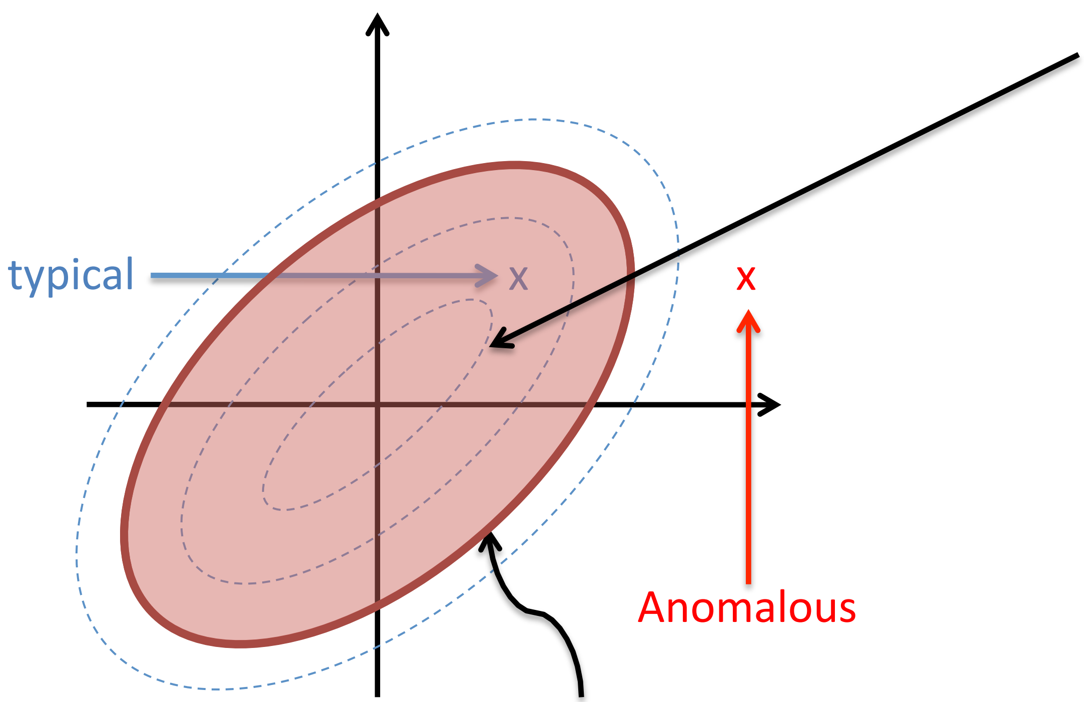
Volume of ellipsoid: $x^T R^{-1} x \le \eta^2$

-Proxy for missed detection rate

-Minimum volume desired

Computed by:

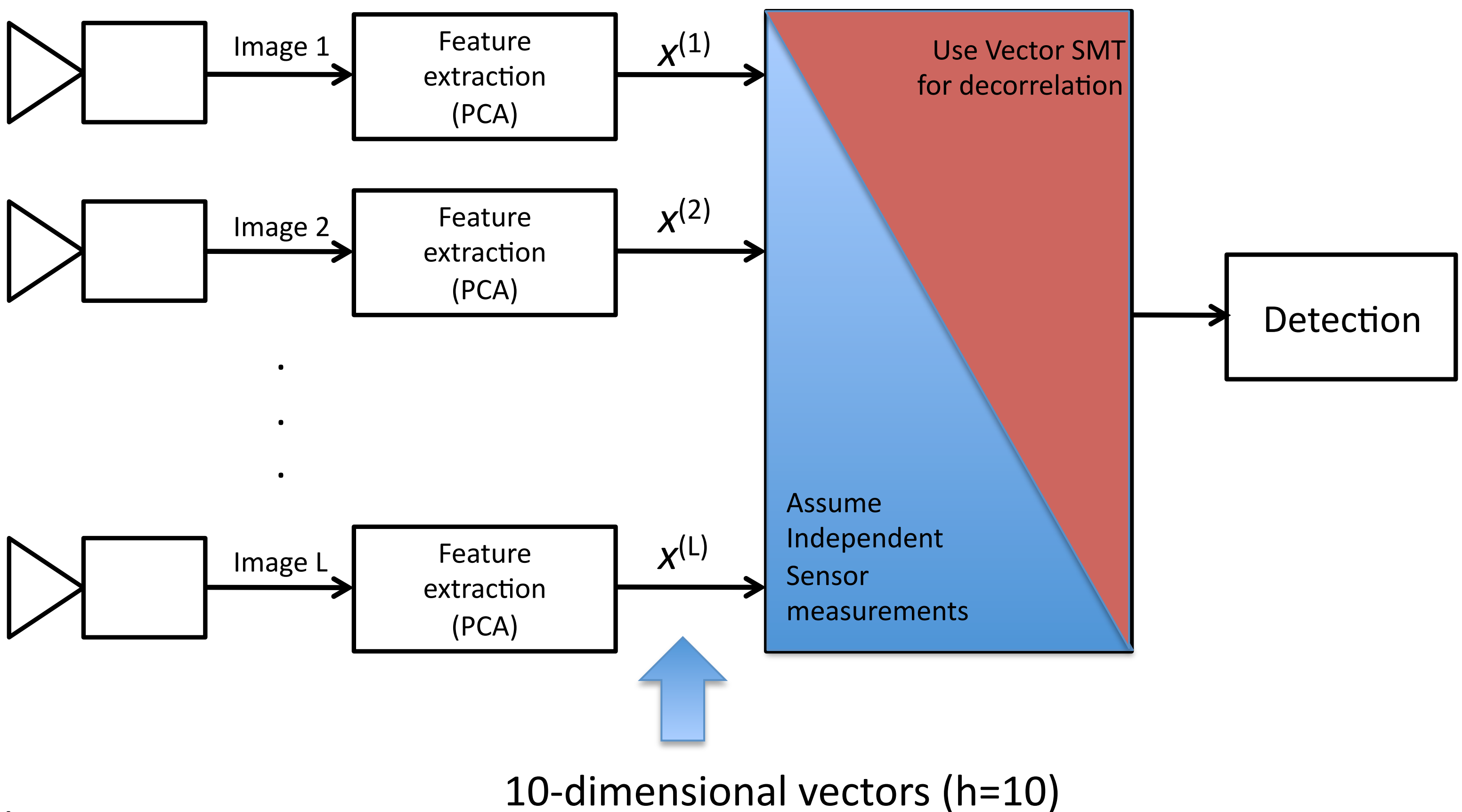
$$V(R,\eta) = \frac{\pi^{\frac{p}{2}}}{\Gamma\left(1 + \frac{p}{2}\right)} \eta^p \sqrt{|R|}$$



$$x^T R^{-1} x = \eta^2$$

η : Controls probability of false alarm

Simulations Setup



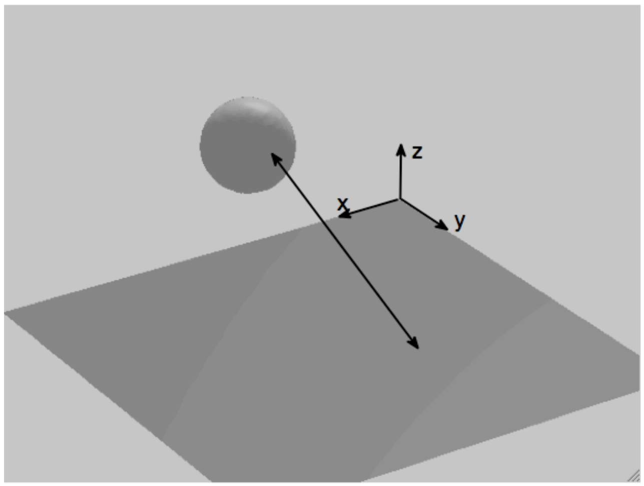
We consider two scenarios:

1 – Assume sensor measurements are independent

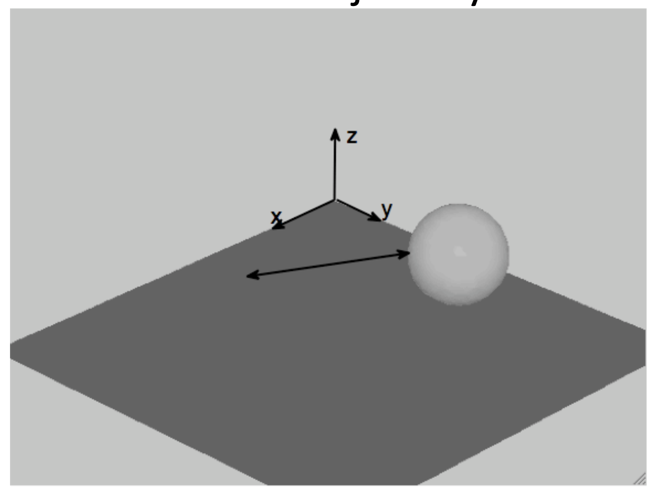
2 – Assume sensor measurements are correlated – use vector SMT for decorrelation

Monitoring a Moving Sphere

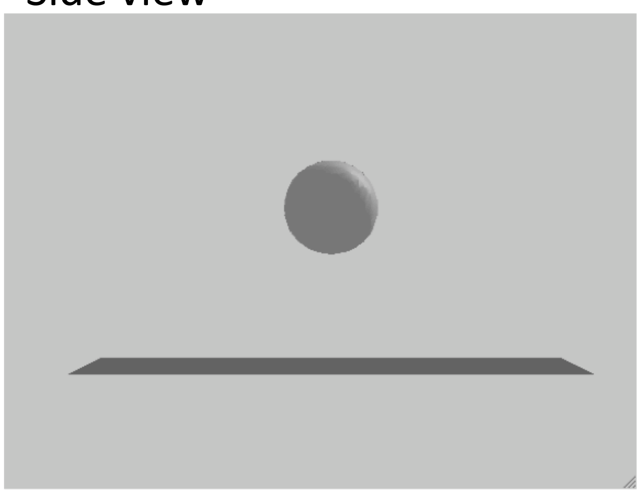
“Typical” trajectory



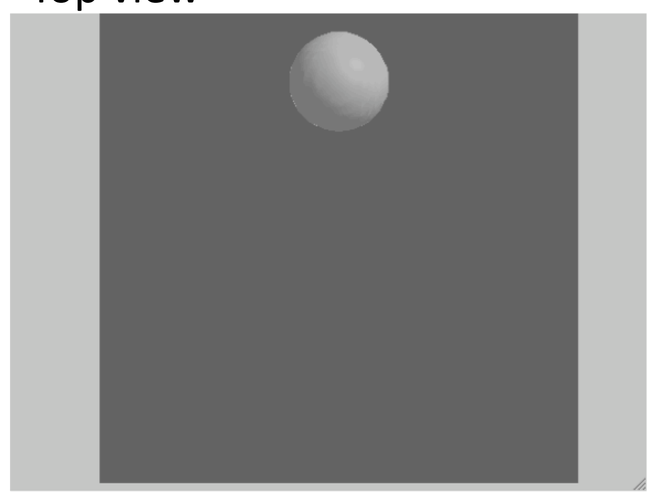
“Anomalous” trajectory



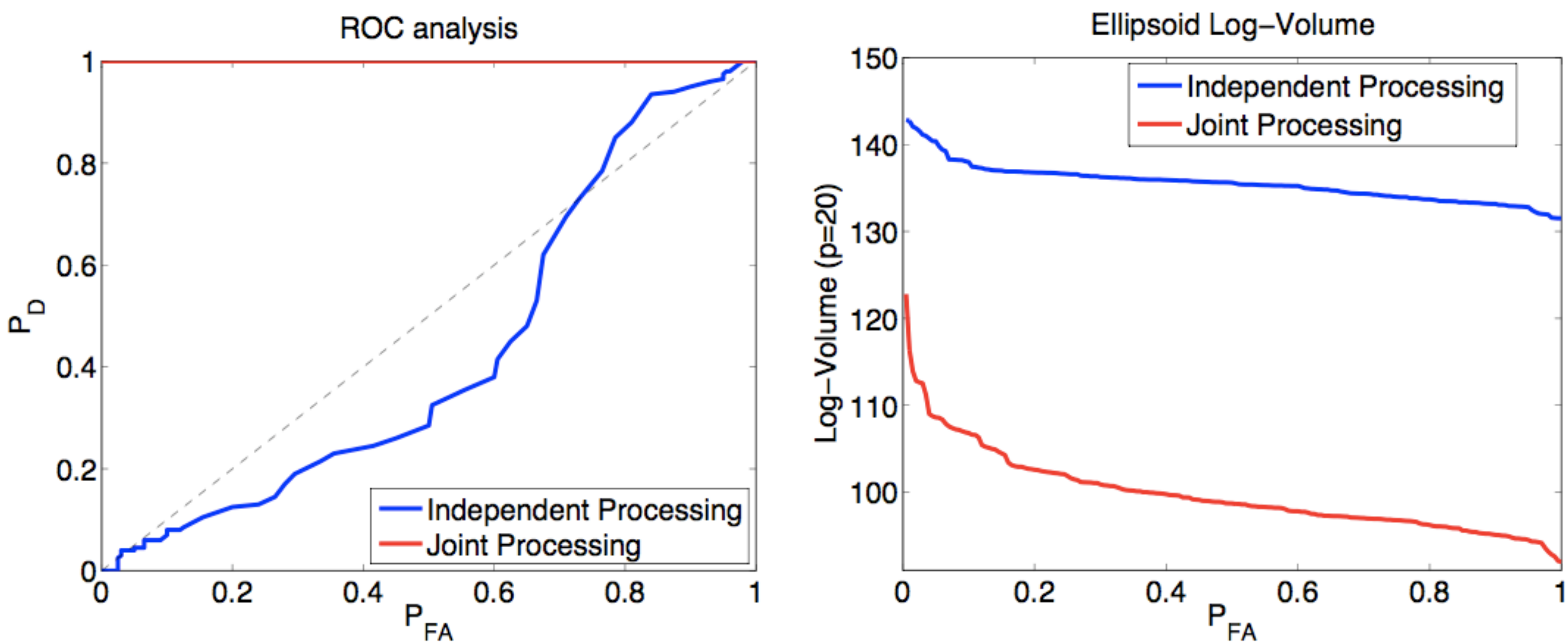
Side view



Top view

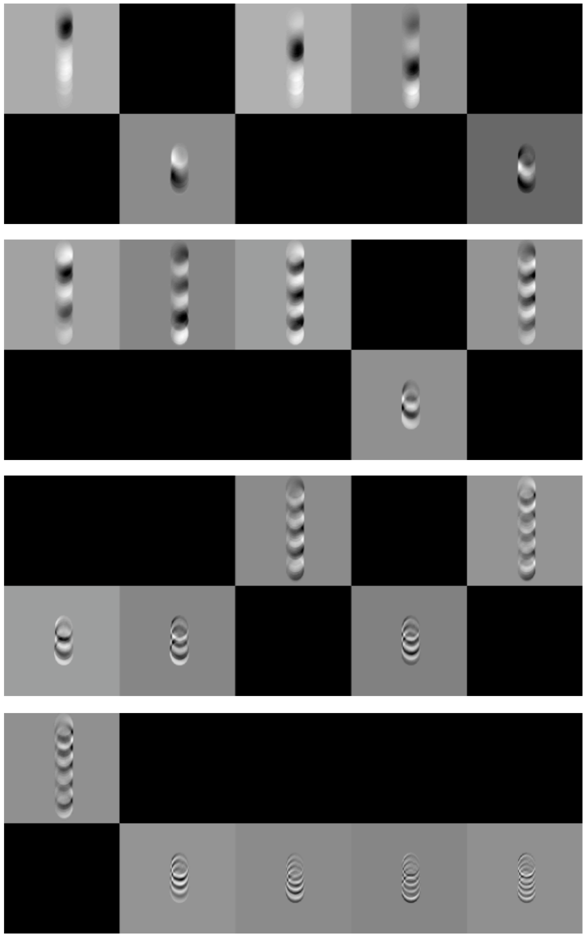


Moving Sphere Anomaly Detection

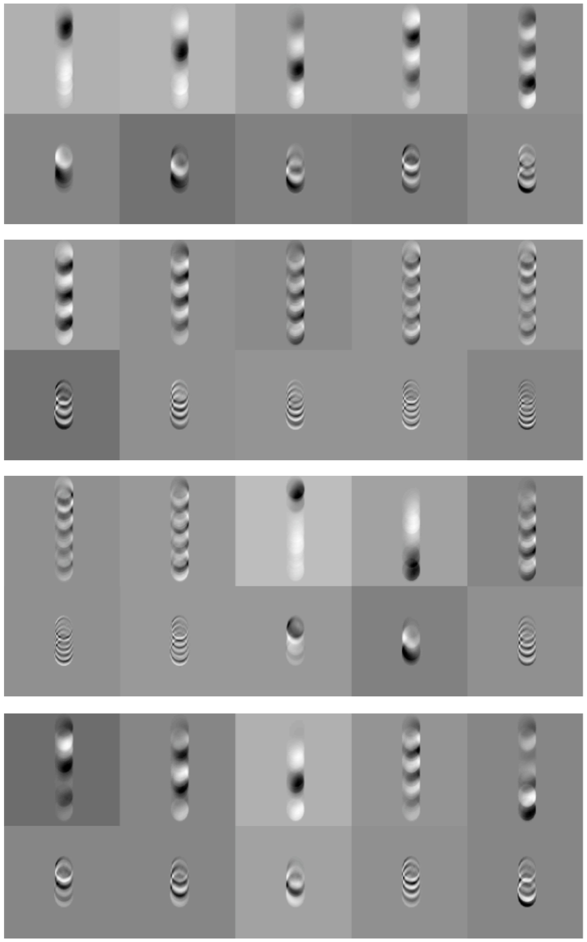


Eigen-Images

Under Independent sensor measurements assumption



Under correlated sensor measurements assumption



Relative Sensitivity of the Two Detectors

Relative sensitivity given by the ratio:

$$\frac{x^t R_2^{-1} x}{x^t R_1^{-1} x}$$

Covariance matrices:

R_1 - sensor independence assumption

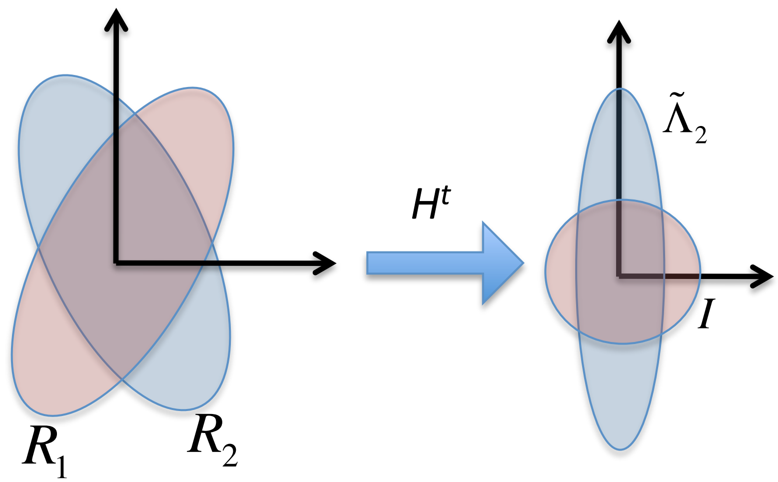
R_2 - correlated sensor measurements assumption

Generalized Eigen-decomposition:

Transform H:

$$HH^t = R_1$$

$$H\tilde{\Lambda}_2 H^t = R_2$$

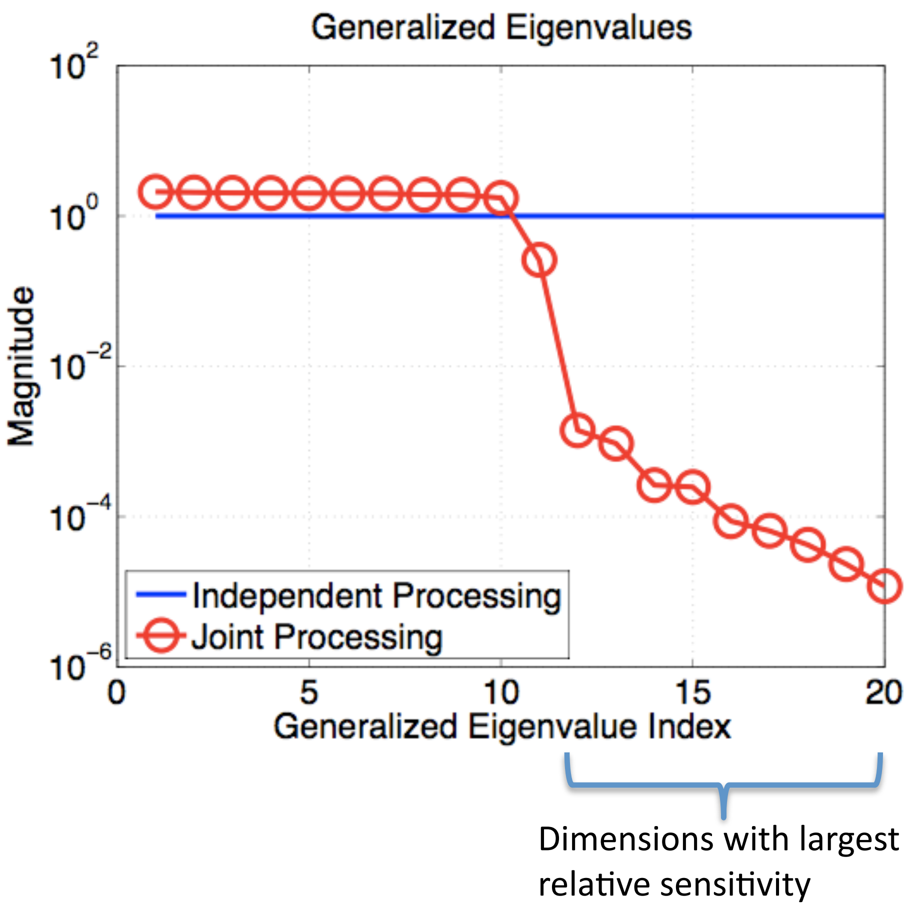


The ratio becomes a weighted sum of independent components:

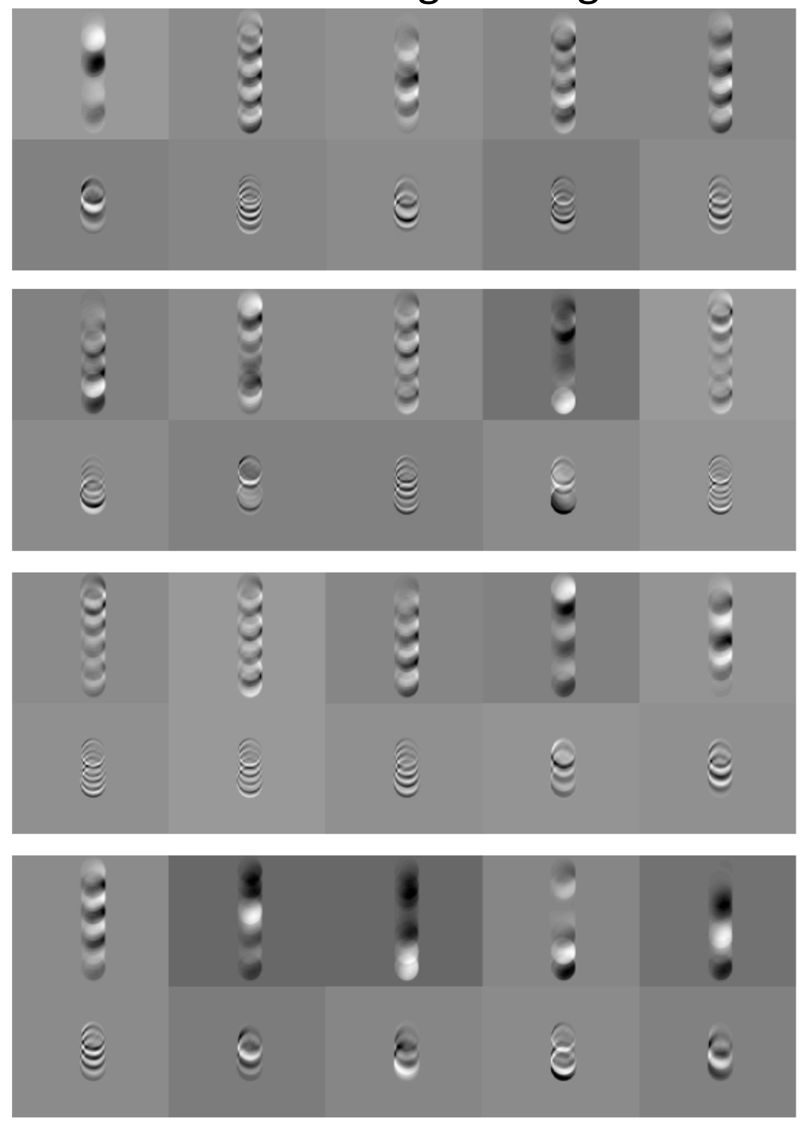
$$\frac{x^t R_2^{-1} x}{x^t R_1^{-1} x} \xrightarrow{\tilde{x} = H^t x} \sum_{k=1}^{p} \frac{1}{[\tilde{\Lambda}_2]_{kk}} \tilde{x}_k^2$$

Relative sensitivity
of generalized coordinate k

Generalized Eigenvalues/Eigen-Images



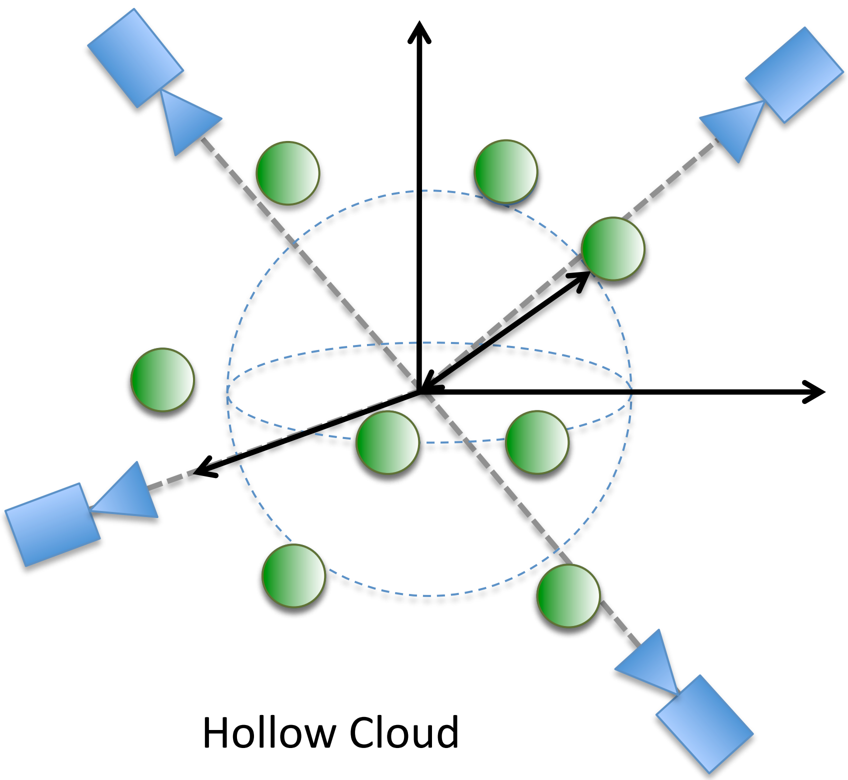
Generalized Eigen-Images



Sphere Clouds

<u>Goal:</u> Monitor the clouds of 30 spheres scene using 14 cameras and decide whether
(1) it is a typical configuration (hollow cloud)
(2) OR it is an anomalous configuration (dense cloud)

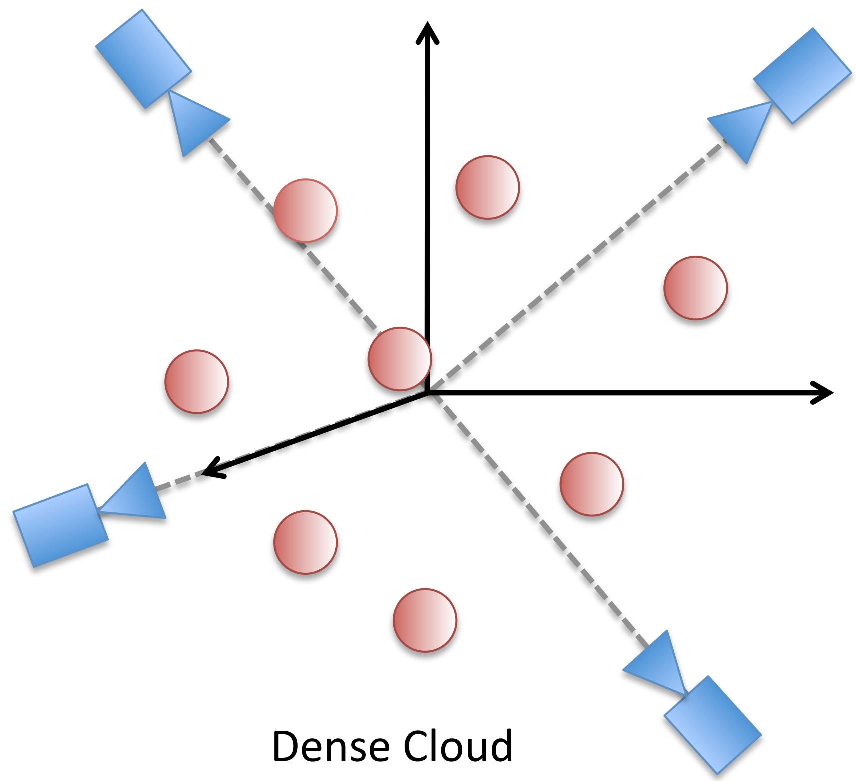
***Typical* Configuration**



Hollow Cloud

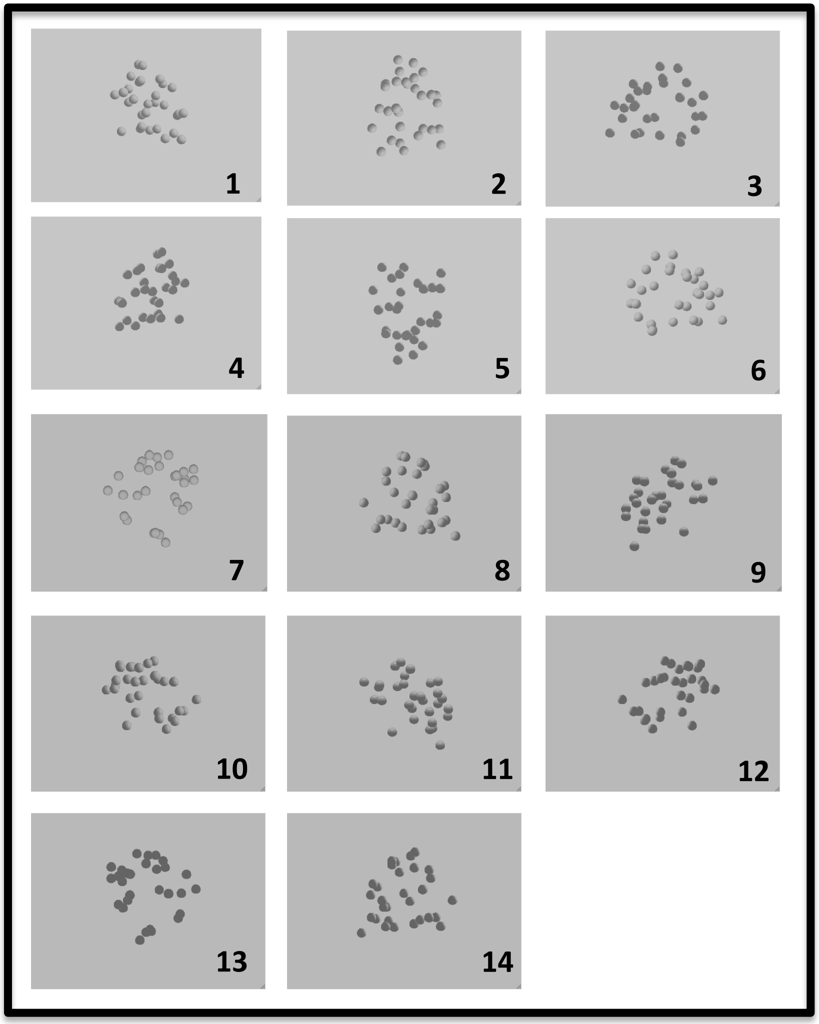
OR

***Anomalous* Configuration**

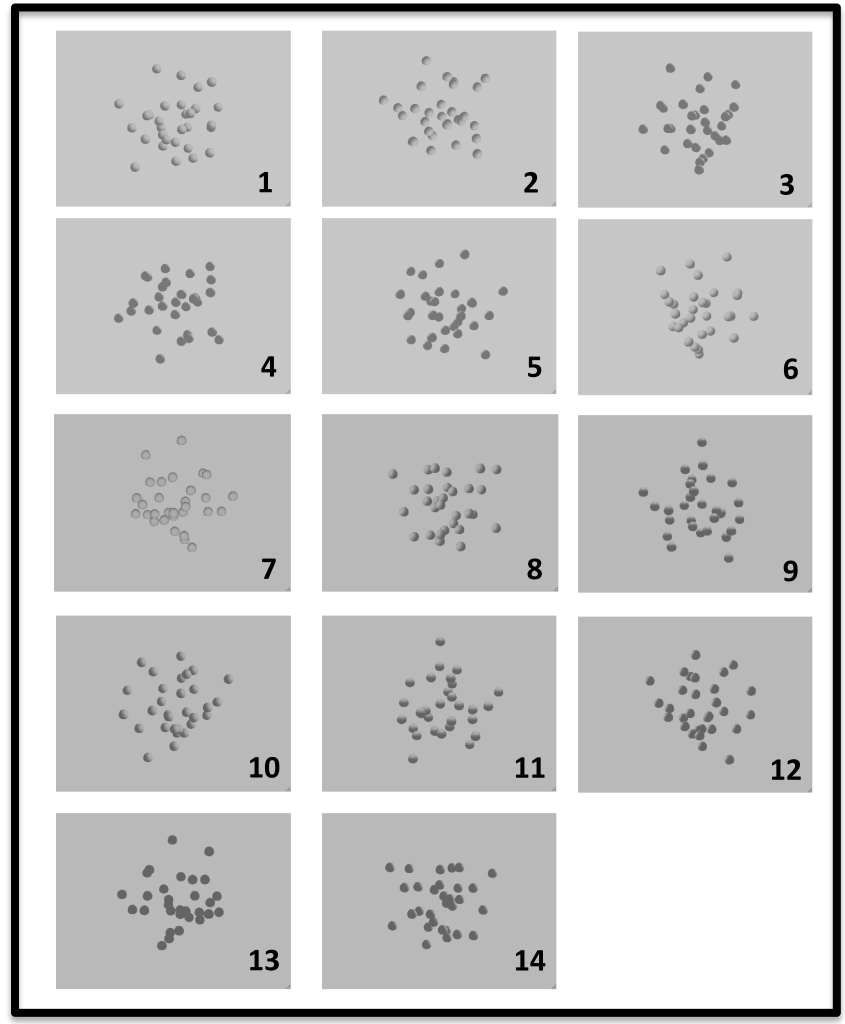


Dense Cloud

Cloud Sample – Camera Views 1-14

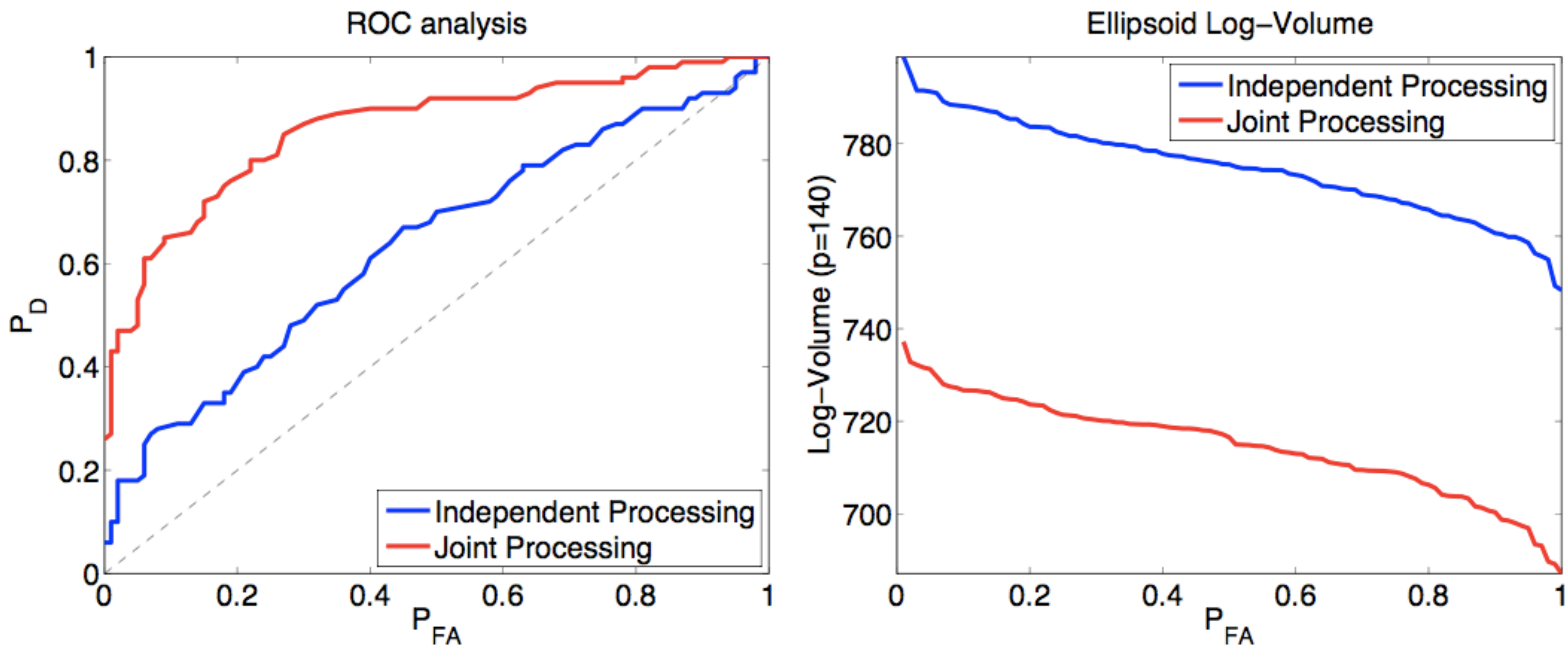


Sample with typical configuration:
Hollow cloud



Sample with anomalous configuration:
Dense cloud

Sphere Cloud - Detection



Conclusions and Future Work

- Vector SMT framework
  - Based on the SMT
  - Decorrelates vector measurements across multiple sensors in a WSN
  - One pair of sensors per iteration
- Simulation results suggest
  - Great potential for use in distributed monitoring applications
  - Multi-view detection of visual anomalies
- Future
  - Analysis of communication costs
  - Comparison with other methods

Thank You