Introduction to MPI

Topics to be covered

- MPI vs shared memory
- Initializing MPI
- MPI concepts -- communicators, processes, ranks
- MPI functions to manipulate these
- Timing functions
- Barriers and the reduction collective operation

Shared and distributed memory

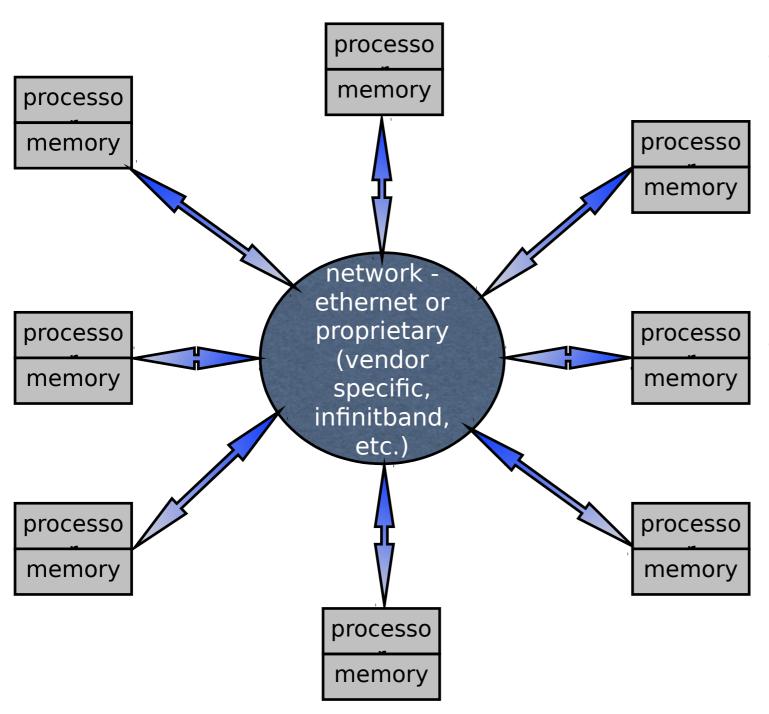
Shared memory

- automatically maintained a consistent image of memory according to some memory model
- fine grained communication possible via loads, stores, and cache coherence
- model and multicore hardware support well aligned
- Programs can be converted piece-wise

Shared and distributed memory

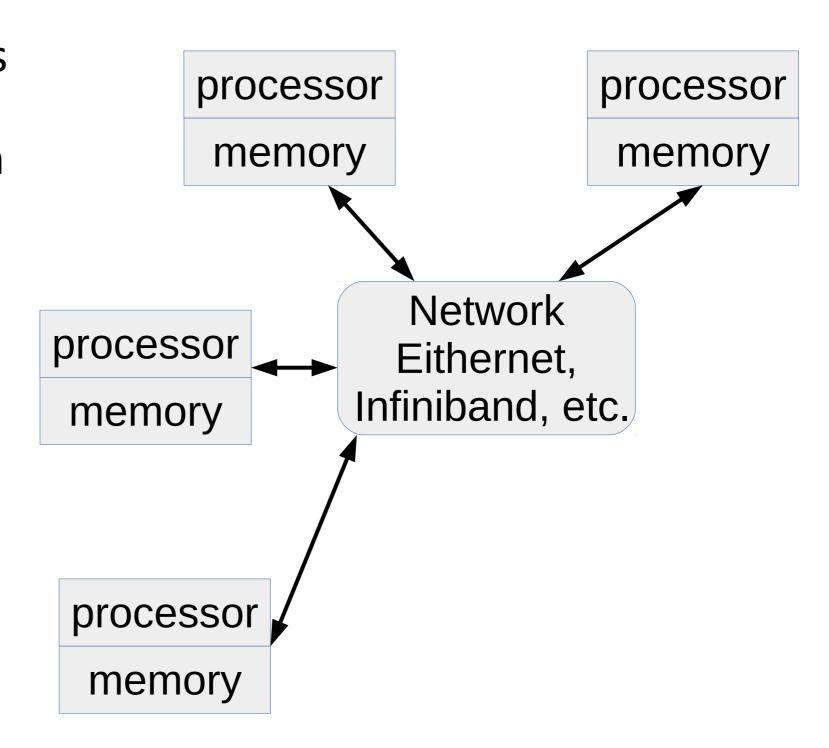
Distributed memory

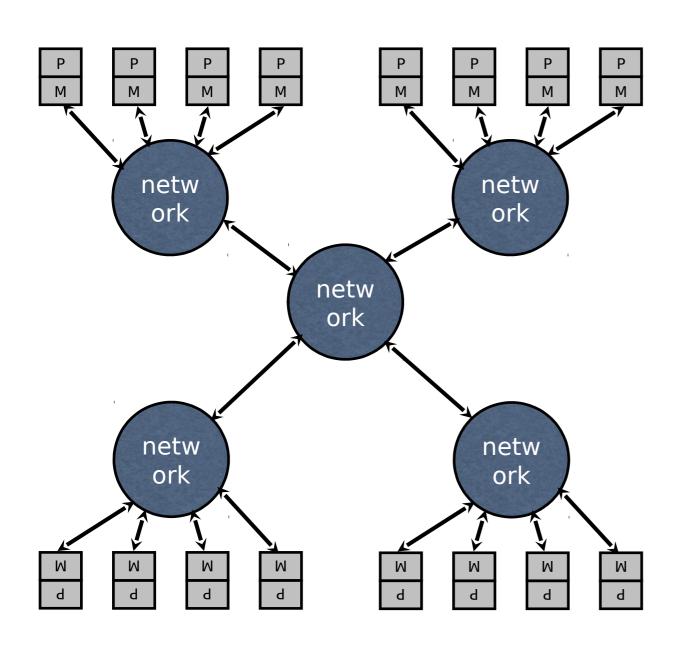
- Program executes as a collection of processes, all communication between processors explicitly specified by the programmer
- Fine grained communication in general too expensive -- programmer must aggregate communication
- Conversion of programs is all-ornothing
- Cost scaling of machines is better than with shared memory -- well aligned with economics of commodity rack mounted blades



- This drawing implies that all processor are equidistant from one another
- This is often not the case -- the network topology and multicores make some processors closer than others
- programmers
 have to exploit

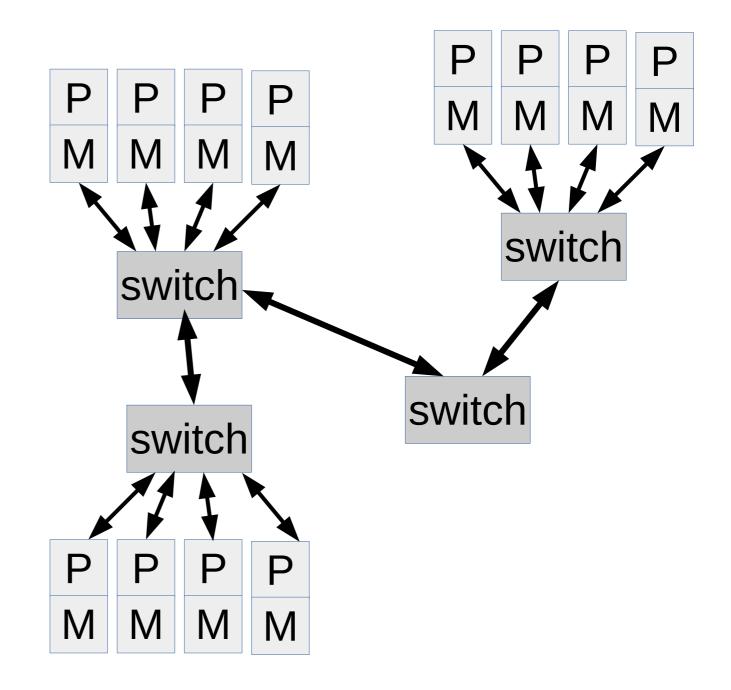
- This drawing implies that all processor are equidistant from one another
- This is often not the case -- the network topology and multicores make some processors closer than others
- programmers have to exploit this manually





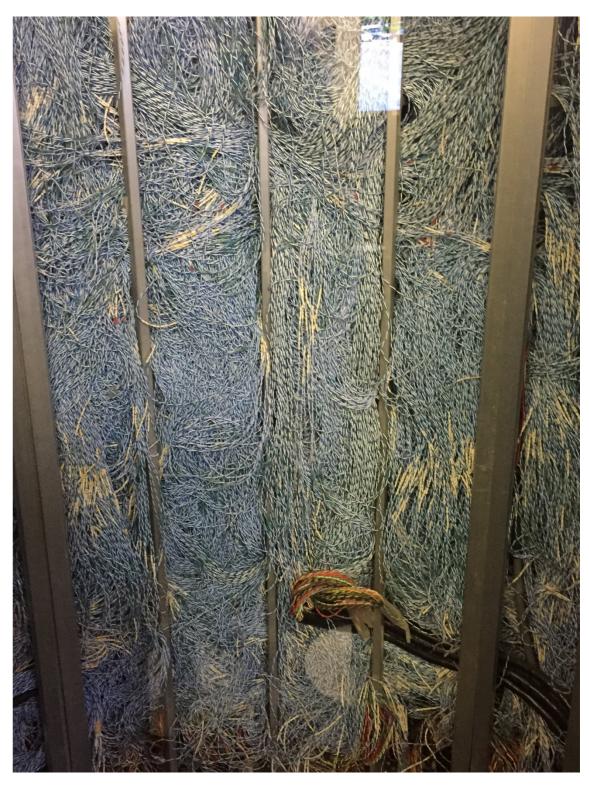
- In reality, processes run on cores, and are closer to other processes on the same processor
- Across processors, some can be reached via a single hop on the network, others require multiple hops
- Not a big issue on small (several hundred processors), but it needs to be considered on large machines.

- In reality, processes run on cores, and are closer to other processes on the same processor
- Across processors, some can be reached via a single hop on the network, others require multiple hops
- Not a big issue on small (several hundred processors), but it needs to be considered on large machines.



Cray-1 80 mhz, 138 - 250 MPFLOPs





Some Seymour Cray quotes

If you were plowing a field, which would you rather use? Two strong oxen or 1024 chickens?

Anybody can build a fast CPU, the trick is to build a fast system.

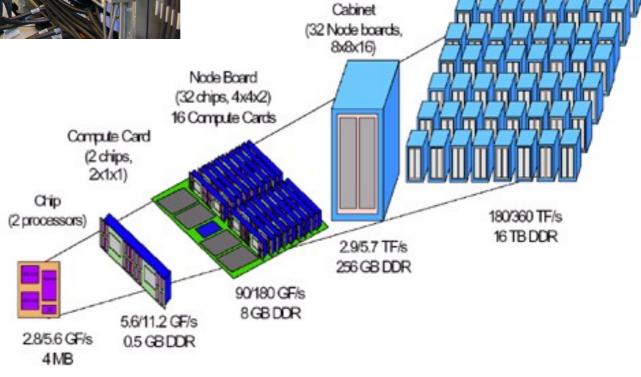
As long as we can make them smaller, we can make them faster.

Parity is for farmers.

131,072 cores BG/L (5.6 GFLOPS)







System

Tianhe-2, 40,960 processors, 10,649,600 cores, 33.9 PFLOPS

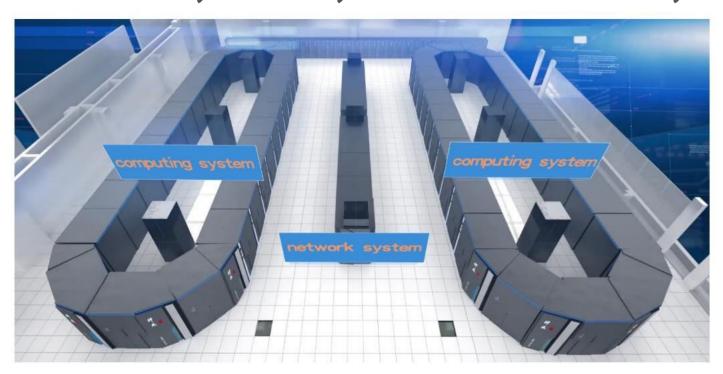
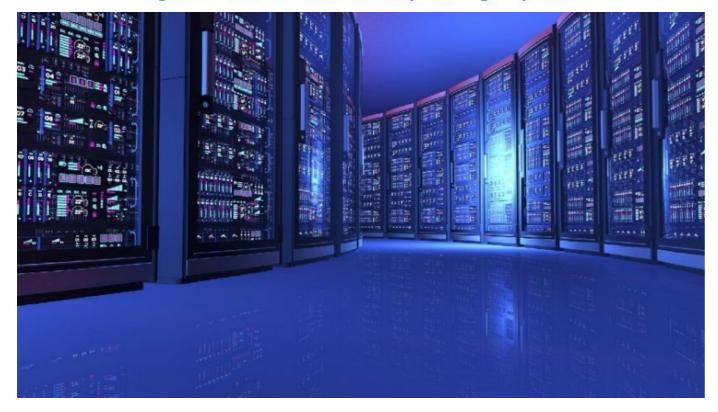


Figure 4: Overview of the Sunway TaihuLight System



TaihuLight has 125 PFLOPS peak performance, 93 PFLOPS on Linpack.

Why use message passing

- Allows control over data layout, locality and communication -- very important on large machines
- Portable across all machines including shared memory machines -- it's a universal parallel programming model.
 Sometimes called the assembly language of paralle programming
- Easier to write deterministic programs
 - simplifies debugging
 - easier to understand programs
- Style needed for efficient messages can lead to better performance than shared memory programs, even on shared memory systems.

Why not use it?

- All or nothing program development generally need to make the entire program parallel to make any part parallel
- Information needed for messages low-level and sometimes hard to program
- Subtle bugs in message passing code can lead to performance problems and deadlock
- Message passing code disrupts the flow of algorithms

SPMD execution is often used with MPI

- Single Program Multiple Data
- Multiple copies of the same program operating on different parts of the data (typically different sections of an array)
- Each program copy executes in a process
- Different processes can execute different paths through the program

SPMD execution

```
for (i=0; i <= (n-1)/2; i++) {
    a[i] = i + 1;
}
for (i=0, i <= n-1; i++) {
    ... = a[i-1];
}
```

for $(i=0; i \le n-1; i++)$
a[i] = i + 1;
}
for $(i=0, i \le n-1; i++)$
= a[i-1];
}

0	1	• • •	n/2-1	n/2
1	2	• • •	49	50

Global index

Local index

0	1	•••	n/2-1	n/2
1	2	•••	49	50

```
for (i=0; i <= n-1; i++) { // n = 100

a[i] = i + 1;

for (i=0, i <= n-1; i++) { for (i=0, i <= n-1; i++) { for (i=0, i <= n-1; i++) } for (i=0, i <= n-1; i++) { for (i=0, i <= n-1; i++) }
```

Work is done by processes

- Each process has a unique rank or process id (often called pid in programs) that is set when program begins executing
- The rank does NOT change during the execution of the program
- Each process has a unique identifier (often called *pid*) that is known to the program

Typical program pattern is

Compute communicate communicate communicate ...

An simple MPI program: Radix sort

- Radix sort works well to sort lists of numbers
- Will assume integers have values from 0 to 65,535
- Have N >> 65,535
 numbers to sort

A sequential radix sort

```
for (i=0; i < 65535; i++) {
  sorted[i] = 0;
for (i=0; i < n; i++)
  sorted[data[i]]++;
for (i=0; i<65535; i++) {
  for (j=0; j < sort[i]; j+
+) {
    fprint("%i\n", i);
```

Want to convert to SPMD message passing code

A sequential radix sort

```
for (i=0; i < 65535; i++)
  sorted[i] = 0;
for (i=0; i < n; i++)
  sorted[data[i]]++;
for (i=0; i<65535; i++) {
  for (j=0; j < sort[i]; j+
+) {
    fprint("%i\n", i);
```

Note that data input not shown --this can require some thought

Data often spread across multiple files to accommodate parallel I/O on large problems

Determining a data layout

Process pid = 0

data[0:N/4-1] i,j Sorted[0:65353] Process pid = 1

data[n/4:2*N/4] i,j Sorted[0:65353]

Process pid = 2

data[2*N/4:3*N/4-1] i,j Sorted[0:65353] Process pid = 2

data[3*n/4:N-1] i,j Sorted[0:65353] Global indices are shown. The local indices used on each processor are, for data,

pid*n/4:(pid+1)*n/4-1
For replicated data,
global and local
indices are the same

Change the program to SPMD

```
data[0:N/4-1]
i,j
Sorted[0:65353]
```

```
data[n/4:2*N/4]
i,j
Sorted[0:65353]
```

```
data[2*N/4:3*N/4-1]
i,j
Sorted[0:65353]
```

data[3*n/4:N-1] i,j Sorted[0:65353]

```
all processors execute this (replicated execution) for (i=0; i < 65535; i++) { sorted[i] = 0; }
```

each processor executes N/4 iterations (assume $N \mod 4 = 0$)

```
for (i=0; i < N/4; i++) {
    sorted[data[i]]++;
}
```

this becomes a sum reduction over the sorted arrays on each processor, i.e. communication. This code does not show that yet.

```
for (i=0; i<65535; i++) {
    for (j=0; j < sort[i]; j++) {
        fprint("%i\n", i);
    }}
```

Data management

data[0:N/4-1] i,j Sorted[0:65353] data[n/4:2*N/4] i,j Sorted[0:65353]

data[2*N/4:3*N/4-1] i,j Sorted[0:65353] data[3*n/4:N-1] i,j Sorted[0:65353]

- All declared variables exist within each process
- There is a global and local logical index space for arrays
 - globally, data has N elements pid*N:(pid+1)*N/4-1
 - *locally*, each process has N/4 elements numbered 0:N/4-1 (if $N \mod 4 == 0$, otherwise $\lceil N/4 \rceil$ or $\lfloor N/4 \rfloor$ elements per processors with some processors having more or fewer elements than other processors
 - The concatenation of the local partitions of data arrays forms the global array data
- The array data is *block* distributed over the processors

Data bounds for block

- Two "obvious" ways to compute
- Let n be the array size, P the number processors

First method

- •Let P be the number of processes, n the number of array elements, $0 \le p \le P-1$ is a process id
- $r = n \mod P$, r = 0, all blocks are the same size, otherwise, first r blocks have $\lceil n/P \rceil$ elements, last P-r have $\lfloor n/P \rfloor$ elements
- •First element on a process p is p[n/P] + min(p,r)
- •Last element on process p is $(p+1)\lfloor n/P \rfloor + min(p+1,r) 1$
- •process with element i is $min(\lfloor i/(\lfloor n/P \rfloor + 1) \rfloor, \lfloor i-r) / \lfloor n/P \rfloor \rfloor)$
- •Example -- 12 elements over 5 processors, $2 = 12 \mod 5$



• Example -- 12 elements over 7 processors, $5 = 12 \mod 7$



Second method

- First element controlled (or owned) by process p is $\lfloor p n/P \rfloor$ (first element and first process id p is 0
- Last element controlled by process p is one less that the first element controlled by process p+1 (the next process)

$$\lfloor (p+1) n/P \rfloor - 1$$

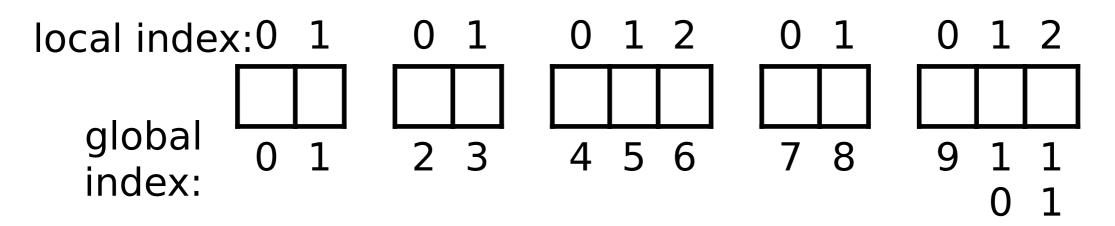
- Process controlling element i is $\lfloor (P(i+1)-1)/n \rfloor$
- Example -- 12 elements over 5 processors, $r = 2 = 12 \mod 5$



• Example -- 17 elements over 5 processors, $r = 2 = 17 \mod 5$

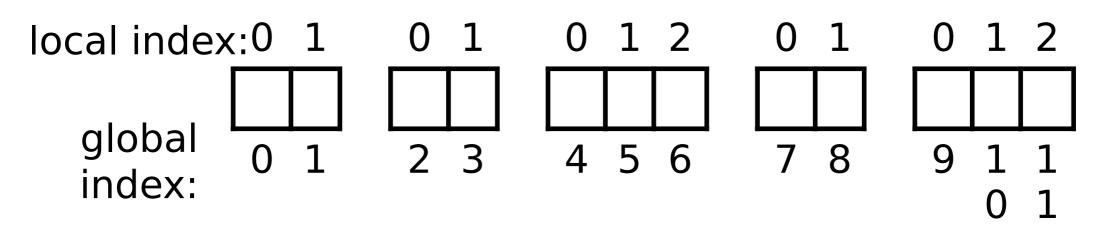


Global vs local indices



- Each part of an array within a process must be indexed as a local element of that array using the local index.
- Logically, each local element is a part of the global array, and within the problem domain has a global index
- It is the MPI *programmer's* responsibility (that means you) to maintain that mapping.

Use macros to access bounds



- Macros or functions can be used to compute these.
- Block lower bound: LB(pid, P, n) = (pid*n/P)
- Block upper bound: UB(pid, P, n) = LB(pid+1, P, n)-1
- Block size: LB(pid+1, P, n) LB(pid, P, n) + 1
- Block owner: Owner(i, P, n) = (P*(i+1)-1)/n

Comparison of the two methods

Operations	First Method	Second Method
Low index	4	2
High index	6	4
Owner	7	4

Assumes floor is free (as it is with integer division although integer division itself may be expensive)

The cyclic distribution

 P0
 P1
 P2
 P3

 Data[0:N:4]
 Data[1:N:4]
 Data[2:N:4]
 Data[3:N:4]

 I,j
 I,j
 I,j
 I,j

 Sorted[0:65353
 Sorted[0:65353
 Sorted[0:65353
 Sorted[0:65353

- Let A be an array with N elements.
- Let the array be cyclically distributed over P processes
- Process p gets elements p, p+P, p+2*P, p+3*P, ...
- In the above
 - process 0 gets elements 0, 4, 8, 12, ... of data
 - process 1 gets elements 1, 5, 9, 13, ... of data
 - process 2 gets elements 2, 6, 10, 14, ... of data
 - process 3 gets elements 3, 7, 11, 15, ... of data

The block-cyclic distribution

- Let A be an array with N elements
- Let the array be block-cyclically distributed over P processes, with blocksize B
- Block b, b = 0 ..., on process p gets elements b*B*P+p*B: b*B*P + (p+1)*B)-1 elements
- With P=4, *B*=3
 - process 0 gets elements [0:2], [12:14], [24:26] of data
 - process 1 gets elements [3:5], [15:17], [27:29] of data
 - process 2 gets elements [6:8], [18:20],[30:32] of data
 - process 3 gets elements [9:11], [21:23],[33:35] of data

System initialization

```
#include <mpi.h> /* MPI library prototypes, etc. */
#include <stdio.h>
// all processors execute this (replicated execution)
int main(int argc, char * argv[]) {
int pid; /* MPI process ID)
int numP; /* number of MPI processes */
int N;
extractArgv(&N, argv); // get N from the arg vector
int sorted[65536]; int data[N/4];
MPI INIT(&argc, &argv); // argc and argv need to be passed in
for (i=0; i < 65535; i++) {
sorted[i] = 0;
                                       data[pid*n/4:pid*N/4-1]
                                          Sorted[0:65353]
```

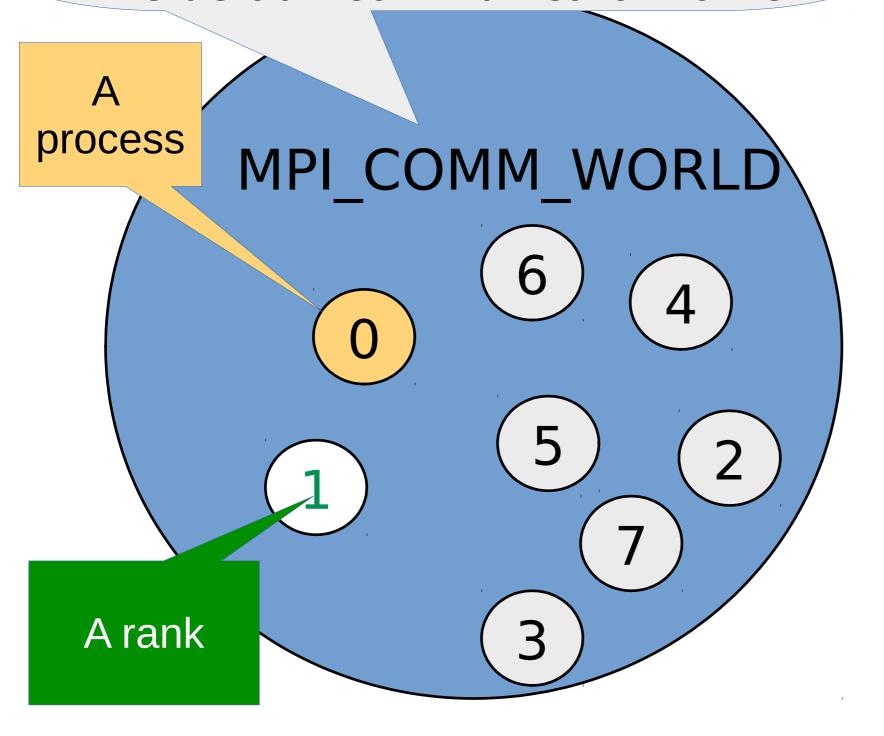
MPI_INIT

- Initialize the MPI runtime
- Does not have to be the first executable statement in the program, but it must be the first MPI call made
- Initializes the default MPI communicator (MPI_COMM_WORLD which includes all processes)
- Reads standard files and environment variables to get information about the system the program will execute on
 - e.g. what machines executes the program?

The MPI environment

The communicator name

(MPI_COMM_WORLD is
the default communicator name



A communicator defines a universe of processes that can exchange messages

Include files

P0 Data[0:N:4] I,j Sorted[0:65353 P1 Data[1:N:4]

Sorted[0:65353

P2

Data[1:N:4]

Sorted[0:65353

P3

Data[1:N:4]

l,j Sorted[0:65353

#include <mpi.h> /* MPI library prototypes, etc. */
#include <stdio.h>

using mpi // Fortran 90 include "mpi.h" // Fortran 77

These may not be shown on later slides to make room for more interesting stuff

Communicator and process info

P0 P1 P2 P3

data[0:N/4-1] data[n/4:2*N/4] data[2*N/4:3*N/4-1] data[3*n/4-1]

data[0:N/4-1] i,j Sorted[0:65353]

```
data[n/4:2*N/4]
i,j
Sorted[0:65353]
```

```
data[2*N/4:3*N/4-1]
i,j
Sorted[0:65353]
```

data[3*n/4:N-1] i,j Sorted[0:65353]

```
// all processors execute this (replicated execution)
int main(int argc, char * argv[]) {
  int pid; /* MPI process ID)
  int numP; /* number of MPI processes */
  int N;
  int lb = LB(pid, numP, N); int ub = UB(pid,numP,N);
  extractArgv(&N, argv);
  int sorted[65536]; int *data;
  MPI INIT(&argc, &argv);
  MPI Comm size(MPI COMM WORLD, &numP);
  for (i=0; i < 65535; i++)
    sorted[i] = 0;
```

Getting the pid for each process

P0 P1 P2 P3 data[0:N/4-1] data[n/4:2*N/4] data[2*N/4:3*N/4-1] data[3*n/4

I,J Sorted[0:65353] i,j Sorted[0:65353] data[2*N/4:3*N/4-1] i,j Sorted[0:65353] data[3*n/4:N-1] i,j Sorted[0:65353]

```
// all processors execute this (replicated execution)
int main(int argc, char * argv[]) {
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  int numP; /* number of MPI processes */
  int N;
  int lb = LB(pid, numP, N); int ub = UB(pid,numP,N);
  extractArgv(&N, argv);
  int sorted[65536]; int* data;
  MPI INIT(&argc, &argv);
  MPI Comm size(MPI COMM WORLD, &numP);
  MPI Comm rank(MPI COMM WORLD, &pid);
  for (i=0; i < 65535; i++)
    sorted[i] = 0;
```

Getting the pid for each process

P0 P1 P2 P3 data[0:N/4-1] data[n/4:2*N/4] data[2*N/4:3*N/4-1] data[3*n/4

I,J Sorted[0:65353] i,j Sorted[0:65353] data[2*N/4:3*N/4-1] i,j Sorted[0:65353] data[3*n/4:N-1] i,j Sorted[0:65353]

```
// all processors execute this (replicated execution)
int main(int argc, char * argv[]) {
  int pid; /* MPI process ID)
  int numP; /* number of MPI processes */
  int N;
  int lb = LB(pid, numP, N); int ub = UB(pid,numP,N);
  extractArgv(&N, argv);
  int sorted[65536]; int* data;
  MPI INIT(&argc, &argv);
  MPI Comm size(MPI COMM WORLD, &numP);
  MPI Comm rank(MPI COMM WORLD, &pid);
  for (i=0; i < 65535; i++)
    sorted[i] = 0;
```

Allocating local storage

P3

data[3*n/4:N-1]

Sorted[0:65353]

```
P0
                          P1
                                             P2
                                      data[2*N/4:3*N/4-1]
                   data[n/4:2*N/4]
 data[0:N/4-1]
                                       Sorted[0:65353]
                   Sorted[0:65353]
Sorted[0:65353]
int main(int argc, char * argv[]) {
  int pid; /* MPI process ID)
  int numP; /* number of MPI processes */
  int N;
  int lb = LB(pid, numP, N); int ub = UB(pid,numP,N);
  extractArgv(&N, argv);
  int sorted[65536]; int* data;
  MPI INIT(&argc, &argv);
  MPI Comm size(MPI COMM WORLD, &numP);
  MPI Comm rank(MPI COMM WORLD, &pid);
  Lb = LB(pid, numP, N); ub = LB(pid, numP, N)-1;
  data = malloc(sizeof(int)*(ub-lb+1)
  for (i=0; i < 65535; i++) {
    sorted[i] = 0;
```

Terminating the MPI program

P2

P3

Data[1:N:4]

Sorted[0:65353

```
Data[0:N:4]
                      Data[1:N:4]
                                          Data[1:N:4]
Sorted[0:65353
                    Sorted[0:65353
                                        Sorted[0:65353
int main(int argc, char * argv[]) {
  int pid; /* MPI process ID)
  int numP; /* number of MPI processes */
  int N;
  int lb = LB(pid, numP, N); int ub = UB(pid,numP,N);
  extractArgv(&N, argv);
  int sorted[65536]; int* data;
  MPI INIT(&argc, &argv);
  MPI Comm size(MPI COMM WORLD, &numP);
  MPI Comm rank(MPI COMM WORLD, &pid);
  Lb = LB(pid, numP, N); ub = LB(pid, numP, N)-1;
  data = malloc(sizeof(int)*(ub-lb+1)
  for (i=0; i < 65535; i++)
    sorted[i] = 0;
```

P0

MPI Finalize();

Time to do something useful

P3

Data[1:N:4]

Sorted[0:65353

P0 P1 P2 Data[0:N:4] Data[1:N:4] Data[1:N:4] Sorted[0:65353 Sorted[0:65353 Sorted[0:65353 int main(int argc, char * argv[]) { int pid; /* MPI process ID) int numP; /* number of MPI processes */ int N; int lb = LB(pid, numP, N); int ub = UB(pid,numP,N); extractArgv(&N, argv); int sorted[65536]; int* data; MPI INIT(&argc, &argv); MPI Comm size(MPI COMM WORLD, &numP); MPI Comm rank(MPI COMM WORLD, &pid); Lb = LB(pid, numP, N); ub = LB(pid, numP, N)-1;data = malloc(sizeof(int)*(ub-lb+1) for (i=0; i < 65535; i++)sorted[i] = 0;sort(data, sort, ub-lb+1);

MPI Finalize();}

The sequential radix sort

```
void sort (sort[], data[], int N) {
  for (i=0; i < N; i++)
    sorted[data[i]]++;
 for (i=0; i<65535; i++) {
    for (j=0; j < sort[i]; j++) {
      fprint("%i\n", i);
```

The parallel radix sort

```
void sort (sort[], data[], int localN) {
  for (i=0; i < N; i++)
    sorted[data[i]]++;
  // pid == 0 only has its results! We
  // need to combine the results here.
 If (pid == 0)
   for (i=0; i<65535; i++) {
      for (j=0; j < sort[i]; j++) {
        fprint("\%i\n", i);
```

Each process sorts the local N elements that it owns. The results from each process need to be combined and sent to a single process for printing, say, the process with *pid==0*.

MPI_Reduce(...)

```
MPI Reduce(
 void *opnd, // data to be reduced
 void *result, // result of the reduction
 int count, // # of elements to be reduced
 MPI Datatype type, // type of the elements
                      // being reduced
 MPI Operator op, // reduction operation
 int root, // pid of the process getting the
          // result of the reduction
 MPI Comm comm // communicator over
                    // which the reduction is
                    // performed
```

MPI_Datatype

Defined as constants in the mpi.h header file

Types supported are

MPI CHAR MPI DOUBLE

MPI_FLOAT MPI_INT

MPI LONG MPI LONG DOUBLE

MPI_SHORT MPI_UNSIGNED_CHAR

MPI_UNSIGNED MPI_UNSIGNED_LONG

MPI_UNSIGNED_SHORT

MPI_Datatype

Defined as constants in the mpi.h header file

Types supported are

```
MPI CHAR
MPI FLOAT
MPI LONG
MPI SHORT
MPI UNSIGNED
MPI UNSIGNED SHORT
MPI DOUBLE
MPI INT
MPI LONGDOUBLE
MPI UNSIGNED CHAR
MPI UNSIGNED LONG
```

MPI_Op

- Defined as constants in the mpi.h header file
- Types supported are

```
MPI BAND
MPI EXOR
MPI LAND
MPI LXOR
MPI MAXLOC
MPI MINLOC
MPI SUM
MPI BOR
MPI BXOR
[MPI LOR
MPI MAX
MPI MIN
MPI PROD
```

sorted, p=0

sorted, p=1

sorted, p=2

sorted, p=3

sorted, p=0

3	5	2	9	8	11	20	4
8	3	6	8	38	5	27	6
1	8	9	0	2	1	2	40
13	15	12	19	18	21	42	3
25	23	39	36	64	38	91	53

MPI_Reduce(MPI_IN_PLACE, sorted, 8, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);

```
P0 res
P0 data
             2
                 3
          1
P1 data
          2
                                 P1 res
                                 P2 res
                                         10
P2 data
          3
              6
                    12
                                 P3 res
             8 12
P3 data
                    16
```

```
20 30
                                P0 res
                                        10
             2
                 3
P0 data
          1
P1 data
          2
                                P1 res
P2 data
          3
             6
                    12
                                P2 res
             8 12
P3 data
                    16
                                P0 res
```

Before reduction After reduction P0 data 10 20 30 P0 data P1 data P1 data P2 data P2 data 12 16 P3 data P3 data

```
MPI_Reduce(MPI_IN_PLACE, data, 3, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);
```

Add the reduction

```
void sort (sort[], data[], int pid, int numP) {
  for (i=0; i < N; i++)
   sorted[data[i]]++;
 // can merge all of the "sorted" arrays here
  if (pid == 0) {
   MPI Reduce(MPI IN PLACE, sorted, 65353, MPI INT,
                MPI SUM, 0, MPI COMM WORLD);
  } else {
   MPI Reduce(sorted, (void *) null, 65353, MPI INT,
               MPI SUM, 0, MPI COMM WORLD);
  // print out the sorted array on process pid==0
Alternatively, could allocate a buffer for final
sorted result. Buffer would be the same size as
sorted.
```

Measure program runtime

```
int main(int argc, char * argv[]) {
double elapsed;
int pid;
int numP;
int N;
MPI Barrier();
elapsed = -MPI Wtime();
sort(data, sort, pid, numP);
elapsed += MPI Wtime( );
if (pid == 0) printSort(final);
MPI Finalize();
```

Wtick() returns a double that holds the number of seconds between clock ticks - 10⁻³ is milliseconds

- MPI_Barrier barrier synchronization
- MPI_Wtick returns the clock resolution in seconds
- MPI_Wtime current time

Wtick() gives the clock resolution

MPI_WTick returns the resolution of MPI_WTime in seconds. That is, it returns, as a double precision value, the number of seconds between successive clock ticks.

double tick = MPI_WTick();

Thus, a millisecond resolution timer will return 10⁻³
This can be used to convert elapsed time to seconds

Sieve of Erosthenes

- Look at block allocations
- Performance tuning
- MPI_Bcast function

Finding prime numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	10 0

To find primes

- 1.start with two, mark all multiples
- 2.find the next unmarked *u* -- it is a prime
- 3.mark all multiples of *u* between k^2 and *n* until k^2 > n
- 4.repeat 2 & 3 until finished

Mark off multiples of primes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	10

To find primes

3 is prime

mark all multiples of 3 > 9

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	10 0

5 is prime

mark all multiples of 5 > 25

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	10 0

7 is prime

mark all multiples of 7 > 49

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	10 0

11 is prime

mark all multiples of 11 > 121

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	10 0

X, 2, 3, 5, 7, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97 are prime.

Want to parallelize this

- Because we are message passing, obvious thing to look at it domain decomposition, i.e. how can we break up the domain being operated on over multiple processors
 - partition data across processors
 - associate tasks with data
- In general, try to find fundamental operations and associate them with data

Find the fundamental operation(s)?

- Marking of the multiples of the last prime found
- if v a multiple of k then v mod k
 == 0
- min-reduction to find the next prime (i.e. smallest unmarked value) across all processes

 broadcast the value to all tasks

```
forall (v = k; v < n+1; v++) {
  if (v mod k != 0) a[v] = 1;
}
```

To make this efficient . . .

- Combine as many tasks as possible onto a single process
- Make the amount of work done by each process similar, i.e. load balance
- Make the communication between tasks efficient

Combining work/data partitioning

- Because processes work on data that they own (the owners compute rule, Rogers and Pingali), the two problems are tightly inter-related.
- Each element is owned by a process
- It is the process that owns the consistent, i.e., up-to-date value of a variable
- All updates to the variable are made by the owner
- All requests for the value of the variable are to the owner

Combining work/data partitioning

- Because processes update the data that they own
- Cyclic distributions have the property that for all elements i on some process p, i mod p = c holds, where c is some integer value
 - Although cyclic usually gives better load balance, it doesn't in this case
 - Lesson -- don't apply rules-of-thumb blindly
- Block, in this case, gives a better load balance
 - computation of indices will be harder

Interplay of decomposition and implementation

- Decomposition affects how we design the implementation
- More abstract issues of parallelization can affect the implementation
- ullet In the current algorithm, let \varPhi be the highest possible prime
- At most, only first $\sqrt{\Phi}$ values may be used to mark off (sieve) other primes
- if P processes, n elements to a process, then if $n/P > \sqrt{\Phi}$ only elements in p=0 will be used to sieve. This means we only need to look for lowest unmarked elements in p=0 and only p=0 needs to send this out, saving a reduction operation.

Use of block partitioning affects marking

- Can mark j, j+k, j+2k, ... where j is the first prime in the block
- Using the parallel method described in earlier psuedocode, would need to use an expensive mod

for all e in the block if e mod k = 0, mark e

We would like to eliminate this.

Sketch of the algorithm

- 1. Create list of possible primes
- 2. On each process, set k = 2
- 3. Repeat
 - 1. On each process, mark all multiples of k
 - 2. On process θ , find smallest unmarked number u, set k=u
 - 3. On process 0, *broadcast k* to all processes
- 4. Until $k^2 > \Phi$ (the highest possible prime)
- 5. Perform a sum reduction to determine the number of primes

Data layout, primes up to 28

array element

$$i = 0$$
 1 2 3 4 5 6 7 8
 $P=0$ 2 3 4 5 6 7 8 9 10

number being checked for "primeness"

$$i = 0$$
 1 2 3 4 5 6 7 8
 $P=1$ 11 12 13 14 15 16 17 18 19

$$i = 0$$
 1 2 3 4 5 6 7 8
 $P=2 \ | 20 \ | 21 \ | 22 \ | 23 \ | 24 \ | 25 \ | 26 \ | 2 \ | 28$

```
Algorithm 1/4
#include <mpi.h>
#include <math.h>
#include <stdio.h>
#include "MyMPI.h"
#define MIN(a,b) ((a)<(b)?(a):(b))
int main (int argc, char *argv[])
 MPI Init (&argc, &argv);
 MPI Barrier(MPI COMM WORLD);
 elapsed time = -MPI Wtime();
 MPI Comm rank (MPI COMM WORLD, &id);
 MPI Comm size (MPI COMM WORLD, &p);
 if (argc != 2)  {
   if (!id) printf ("Command line: %s <m>\n", argv[0]);
   MPI Finalize(); exit (1);
```

Algorithm, 2/4

```
n = atoi(argv[1]);
low value = 2 + BLOCK LOW(id,p,n-1);
high value = 2 + BLOCK HIGH(id,p,n-1);
size = BLOCK SIZE(id,p,n-1);
proc0 size = (n-1)/p;
if ((2 + proc0 size) < (int) sqrt((double) n)) {
 if (!id) printf ("Too many processes\n");
 MPI Finalize();
                  allocate array
 exit (1);
                     to use to
                   mark primes
marked = (char *) malloc (size);
if (marked == NULL) {
 printf ("Cannot allocate enough memory\n");
 MPI Finalize();
 exit (1);
```

Get min and max possible prime on *p* in global space

Figure out if too many processes for $\sqrt{\Phi}$ candidates on p=0

Block Low

Block hIGH

$$i = 9$$
 10 11 12 13 14 15 16 17
 $P=1$ 11 12 13 14 15 16 17

$$i = 18 \quad 19 \quad 20 \quad 21 \quad 22 \quad 23 \quad 24 \quad 25 \quad 26$$
 $P = 2 \quad 20 \quad 21 \quad 22 \quad 23 \quad 24 \quad 25 \quad 26$

Low value

High value

Algorithm 3/4 (a)

```
for (i = 0; i < size; i++) marked[i] = 0; // initialize marking array
if (!id) index = 0; // p=0 action, find first prime
prime = 2;
do { // prime = 2 first time through, sent by bcast on later iterations
    Find first element to mark on each processor
    Mark that element and every kth element on the processor
    Find the next unmarked element on P0. This is the next prime
    Send that prime to every other processor
} while (prime * prime <= n);</pre>
```

Algorithm 3/4 (b)

```
Initialize array and find first prime
  // Find first element to mark on each procesor
  do \{ // \text{ prime} = 2 \text{ first time through, sent by } bcast \text{ on later iterations} \}
    if (prime * prime > low value) // find first value to mark
      first = prime * prime - low value; // first item in this block
    else {
      if (!(low value % prime)) first = 0; // first element divisible
                                            // by prime
      else first = prime - (low value % prime);
    Find first element to mark on each procesor
    Mark that element and every kth element on the processor
    Find the next unmarked element on P0. This is the next prime
    Send that prime to every other processor
  } while (prime * prime <= n);</pre>
```

Algorithm 3/4 (c)

```
Initialize array and find first prime

do { // prime = 2 first time through, sent by bcast on later iterations

Find first element to mark on each procesor

// Mark that element and every kth element on the processor

for (i = first; i < size; i += prime) marked[i] = 1; // mark every kth item

Find the next unmarked element on P0. This is the next prime

Send that prime to every other processor

} while (prime * prime <= n);
```

Algorithm 3/4 (d)

```
Initialize array and find first prime
 do \{ // \text{ prime} = 2 \text{ first time through, sent by } bcast \text{ on later iterations} \}
     Find first element to mark on each procesor
    Mark that element and every kth element on the processor
    // Find the next unmarked element on P0. This is the next prime
    if (!id) \{ // p = 0 \text{ action, find next prime by finding unmarked element } \}
      while (marked[++index]);
      prime = index + 2;
    // Send that prime to every other processor
   MPI Bcast (&prime, 1, MPI INT, 0, MPI COMM WORLD);
  } while (prime * prime <= n);</pre>
```

Algorithm 3/4 full code

```
for (i = 0; i < \text{size}; i++) marked[i] = 0; // initialize marking array
if (!id) index = 0; // p=0 action, find first prime
prime = 2;
do \{ // \text{ prime} = 2 \text{ first time through, sent by } bcast \text{ on later iterations} \}
 if (prime * prime > low value) // find first value to mark
    first = prime * prime - low value; // first item in this block
 else {
    if (!(low value % prime)) first = 0; // first element divisible by prime
   else first = prime - (low value % prime);
 for (i = first; i < size; i += prime) marked[i] = 1; // mark every kth item
 if (!id) \{ // p=0 \text{ action, find next prime by finding unmarked element } \}
    while (marked[++index]);
    prime = index + 2;
 MPI Bcast (&prime, 1, MPI INT, 0, MPI COMM WORLD);
} while (prime * prime <= n);</pre>
```

First prime index = 0prime = 2

not
$$2 * 2 > 11$$
 local $i = 0$
 1
 2
 3
 4
 5
 6
 7
 8

 11 % $2 == 1$
 first = 2 - (11 % 2)
 P=0
 11
 12
 13
 14
 15
 16
 17
 18
 19

 first = 1

$$not \ 2 * 2 > 20 \ local = 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$$
 $20 \% \ 2 == 0$
 $first = 0$
 $P = 0$
 $20 \ 21 \ 22 \ 23 \ 24 \ 25 \ 26 \ 2 \ 28$

third prime $\frac{index = 3}{prime = 5}$

$$5*5>2 \quad local \ i = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

$$first = 5*5-2 \quad P=0 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$$

$$first = 23$$

$$5*5>20 \quad local = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

$$first = 5*5-20 \quad P=0 \quad 20 \quad 21 \quad 22 \quad 23 \quad 24 \quad 25 \quad 26 \quad 2 \quad 28$$

$$first = 5$$

Mark every prime elements starting with *first* index = 0

prime = 2

$$2*2>4$$
 $local i = 0$ 1 2 3 4 5 6 7 8
 $first = 2*2-2$ $P=0$ 2 3 4 5 6 7 8 9 10

not
$$2 * 2 > 11$$
 local $i = 0$ 1 2 3 4 5 6 7 8
 $11 \% 2 == 1$
first = $2 - (11 \% 2)$ P=0 11 12 13 14 15 16 17 18 19
first = 1

$$not \ 2 * 2 > 20 \ local = 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$$
 $20 \% \ 2 == 0$
 $first = 0$
 $P = 0$
 $20 \ 21 \ 22 \ 23 \ 24 \ 25 \ 26 \ 2 \ 28$

Algorithm 4/4

```
// on each processor count the number of primes, then reduce this total
count = 0;
for (i = 0; i < \text{size}; i++) if (!marked[i]) count++;
MPI Reduce (&count, &global count, 1, MPI INT, MPI SUM,
              0, MPI COMM WORLD);
elapsed time += MPI Wtime();
if (!id) {
 printf ("%d primes are less than or equal to %d\n",
        global count, n);
 printf ("Total elapsed time: %10.6f\n", elapsed time);
MPI Finalize ();
return 0;
```

$$index = 0$$
 $prime = 2$

 $global_count = 1 + 4 + 2$

$$count = 1 P = 0 2 3 4 5 6 7 8 9 10$$

$$count = 2 P = 0 20 21 22 23 24 25 26 27 28$$

Other MPI environment management routines

- MPI_Abort (comm, errorcode)
 - Aborts all processors associated with communicator *comm*
- MPI_Get_processor_name(&name, &length)
 - MPI version of *gethostname*, but what it returns is implementation dependent. *gethostname* may be more portable.
- MPI_Initialized(&flag)
 - Returns **true** if MPI_Init has been called, **false** otherwise

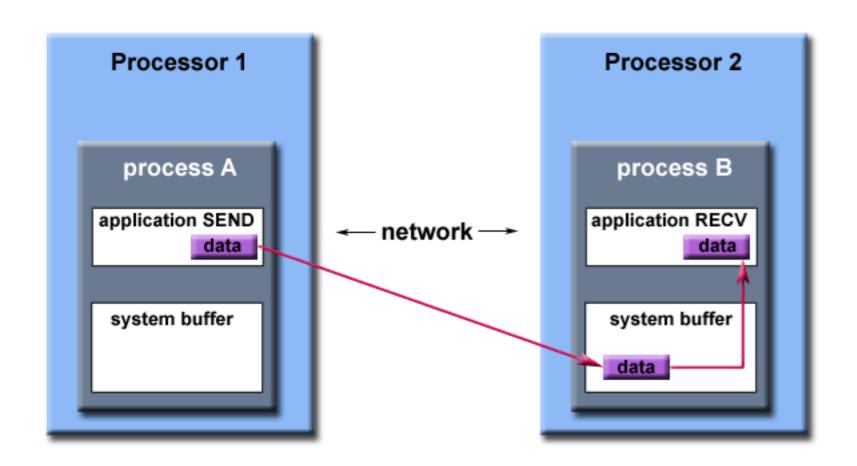
point-to-point communication

- Most MPI communication is between a pair of processors
 - send/receive transmits data from the sending process to the receiving process
- MPI point-to-point communication has many flavors:
 - Synchronous send
 - Blocking send / blocking receive
 - Non-blocking send / non-blocking receive
 - Buffered send
 - Combined send/receive
 - "Ready" send (matching receive already posted.)
- All types of sends can be paired with all types of receive

Buffering

What happens when

- A send occurs before the receiving process is ready for the data
- The data from multiple sends arrive at the receiving task which can only accept one at a time



Path of a message buffered at the receiving process

System buffer space

Not part of the standard -- an "implementation detail

- Managed and controlled by the MPI library
- Finite
- Not well documented -- size maybe a function of install parameters, consequences of running out not well defined
- Both sends and receives can be buffered

Helps performance by enabling asynchronous send/recvs
Can hurt performance because of memory copies
Program variables are called *application buffers* in MPIspeak

Blocking and non-blocking pointto-point communication

Blocking

- Most point-to-point routines have a blocking and nonblocking mode
- A blocking send call returns only when it is safe to modify/reuse the application buffer. Basically the data in the application buffer has been copied into a system buffer or sent.
- Blocking send can be synchronous, which means call to send returns when data is safely delivered to the recv process
- Blocking send can be asynchronous by using a send buffer
- A blocking receive call returns when sent data has arrived and is ready to use

Blocking and non-blocking pointto-point communication

Non-blocking

- Non-blocking send and receive calls behave similarly and return almost immediately.
- Non-blocking operations request the MPI library to perform the operation when it is able. It cannot be predicted when the action will occur.
- You should not modify any application buffer (program variable) used in non-blocking communication until the operation has finished. Wait calls are available to test this.
- Non-blocking communication allows overlap of computation with communication to achieve higher performance

Synchronous and buffered sends and receives

- synchronous send operations block until the receiver begins to receive the data
- buffered send operations allow specification of a buffer used to hold data (this buffer is not the application buffer that is the variable being sent or received)
 - allows user to get around system imposed buffer limits
 - for programs needing large buffers, provides portability
 - One buffer/process allowed
 - synchronous and buffered can be matched

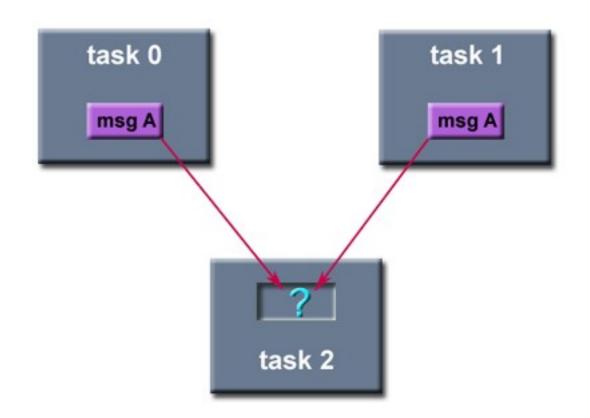
Ordering of messages and fairness

- Messages received in-order
- If a sender sends two messages, (*m1* and *m2*) to the same destination, and both match the same kind of receive, *m1* will be received before *m2*.
- If a receiver posts two receives (*r*1 followed by *r*2), and both are looking for the same kind of messages, *r*1 will receive a message before *r*2.
- Operation starvation is possible
 - task2 performs a single receive. task0 and task3 both send a message to task2 that matches the receive. Only one of the sends will complete if the receive is only executed once.
 - It is the programmer's job to ensure this doesn't happen

Operation starvation

Only one of the *sends* will complete.

Networks are generally not deterministic, cannot be predicted whose message will arrive at *task2* first, and which will complete.



Basic sends and receives

- MPI_send(buffer, count, type, dest, tag, comm)
- MPI_Isend(buffer, count, type, dest, tag, comm, request)
- MIP_Recv(buffer, count, type, source, tag, comm, status)
- MPI_Irecv(buffer, count, type, source, tag, comm, request)

I forms are non-blocking

Basic sends/recv arguments (*I* forms are non-blocking)

- MPI_send(buffer, count, type, dest, tag, comm)
- MPI_Isend(buffer, count, type, dest, tag, comm, request)
- MIP_Recv(buffer, count, type, source, tag, comm, status)
- MPI_Irecv(buffer, count, type, source, tag, comm, request)
- buffer: pointer to the data to be sent or where received (a program variable)
- count: number of data elements of type (*not bytes!*) to be sent
- type: an MPI Type
- tag: the message type, any unsigned integer 0 32767.
- comm: sender and receiver communicator

Basic send/recv arguments

- MPI_send(buffer, count, type, dest, tag, comm)
- MPI_Isend(buffer, count, type, dest, tag, comm, request)
- MIP_Recv(buffer, count, type, source, tag, comm, status)
- MPI_Irecv(buffer, count, type, source, comm, request)
- dest: rank of the receiving process
- source: rank of the sending process
- request: for non-blocking operations, a handle to an MPI_Request structure for the operation to allow *wait* type commands to know what send/recv they are waiting on
- status: the source and tag of the received message. This is a pointer to the structure of type MPI_Status with fields MPI_SOURCE and MPI_TAG.

Blocking send/recv/etc.

MPI_Send: returns after *buf* is free to be reused. Can use a system buffer but not required, and can be implemented by a system send.

MPI_Recv: returns after the requested data is in *buf*.

MPI_Ssend: blocks sender until the application buffer is free and the receiver process started receiving the message

MPI_Bsend: permits the programmer to allocate buffer space instead of relying on system defaults. Otherwise like *MPI_Send*.

MPI_Buffer_attach (&buffer, size): allocate a message buffer with the specified size

MPI_Buffer_detach (&buffer, size): frees the specified buffer

MPI_Rsend: blocking ready send, copies directly to the receive application space buffer, but the receive must be posted before being invoked. Archaie.

MPI_Sendrecv: performs a blocking send and a blocking receive. Processes can swap without deadlock

Example of blocking send/recv

```
#include "mpi.h"
#include <stdio.h>
int main(argc,argv)
int argc;
char *argv[]; {
int numtasks, rank, dest, source, rc, count, tag=1;
char inmsg, outmsg='x';
MPI Status Stat; // status structure
MPI Init(&argc,&argv);
MPI Comm size(MPI COMM WORLD, &numtasks);
MPI Comm rank(MPI COMM WORLD, &rank);
```

Example of blocking send/recv

```
if (rank == 0) {
 dest = 1;
 source = 1;
 rc = MPI Send(&outmsg, 1, MPI CHAR, dest, tag, MPI COMM WORLD);
 rc = MPI Recv(&inmsg, 1, MPI CHAR, source, tag, MPI COMM WORLD,
&Stat);
\} else if (rank == 1) {
 dest = 0;
 source = 0;
 rc = MPI Recv(&inmsg, 1, MPI CHAR, source, tag, MPI COMM WORLD,
&Stat);
 rc = MPI Send(&outmsg, 1, MPI CHAR, dest, tag, MPI COMM WORLD);
rc = MPI Get count(&Stat, MPI CHAR, &count); // returns # of type received
printf("Task %d: Received %d char(s) from task %d with tag %d \n",
      rank, count, Stat.MPI SOURCE, Stat.MPI TAG);
MPI Finalize();
```

Example of blocking send/recv

```
if (rank == 0) {
 dest = 1;
 source = 1;
 rc = MPI \ Send(\&outmsg, 1, MPI \ CHAR, dest, tag, MPI \ COMM \ WORLD);
 rc = MPI Recv(&inmsg, 1, MPI CHAR, source, tag, MPI COMM WORLD,
&Stat);
\} else if (rank == 1) {
 dest = 0;
 source = 0;
 rc = MPI Recv(&inmsg, 1, MPI CHAR, source, tag, MPI COMM WORLD,
&Stat);
 rc = MPI Send(&outmsg, 1, MPI CHAR, dest, tag,
MPI COMM WORLD);
                                     green/italic
 task
                  task
                                    send
                                     blue/bold send
```

Why the reversed send/recv orders?

```
if (rank == 0) {
  dest = 1;
  source = 1;
  rc = MPI_Send(&outmsg, 1, MPI_CHAR, dest, tag, MPI_COMM_WORLD);
  rc = MPI_Recv(&inmsg, 1, MPI_CHAR, source, tag, MPI_COMM_WORLD, &Stat);
} else if (rank == 1) {
  dest = 0;
  source = 0;
  rc = MPI_Recv(&inmsg, 1, MPI_CHAR, source, tag, MPI_COMM_WORLD, &Stat);
  rc = MPI_Send(&outmsg, 1, MPI_CHAR, dest, tag, MPI_COMM_WORLD);
}
```

From stackoverflow

http://stackoverflow.com/questions/20448283/deadlock-with-mpi

MPI_Send may or may not block [until a recv is posted]. It will block until the sender can reuse the sender buffer. Some implementations will return to the caller when the buffer has been sent to a lower communication layer. Some others will return to the caller when there's a matching MPI_Recv() at the other end. So it's up to your MPI implementation whether if this program will deadlock or not.

Non-blocking operations

• MPI_Isend, MPI_Irecv, MPI_Issend, Ibsend, Irsend: similar to MPI_Send, MPI_Recv, MPI_Ssend, Bsend, Rsend except that a Test or Wait must be used to determine that the operation has completed and the buffer may be read (in the case of a recv) or written (in the case of a send)

Wait and probe

MPI_Wait (&request, &status): wait until the operation specified by request (specified in an *Isend/Irecv* finishes)

MPI_Waitany (count, & array_of_requests, & index, & status): wait for any blocking operations specified in & array_of_requests to finish

MPI_Waitall (count, & array_of_requests, & array_of_statuses): wait for all blocking operations specified in & array_of_requests to finish

MPI_Waitsome (incount, &array_of_requests, &outcount, &array_of_statuses): wait for at least one request to finish, the number is returned in *outcount*.

MPI_Probe (source, tag, comm, &status): performs a blocking test but doesn't require a corresponding receive to be posted.

Non-blocking operations

- MPI_Test (&request, &flag,&status)
- MPI_Testany (count, &array_of_requests, &index, &flag, &status)
- MPI_Testall (count,&array_of_requests,&flag, &array_of_statuses)
- MPI_Testsome (incount, &array_of_requests, &outcount, &array_of_offsets, &array_of_statuses)
- Like the wait operations, but do not block

Non-blocking example

```
#include "mpi.h"
#include <stdio.h>
int main(argc,argv)
int argc;
char *argv[]; {
int numtasks, rank, next, prev, buf[2], tag1=1, tag2=2;
MPI Request reqs[4];
MPI Status stats[4];
MPI Init(&argc,&argv);
MPI Comm size(MPI COMM WORLD, &numtasks);
MPI Comm rank(MPI COMM WORLD, &rank);
```

```
Non-blocking example
prev = rank-1;
next = rank+1;
if (rank == 0) prev = numtasks - 1;
if (rank == (numtasks - 1)) next = 0;
MPI Irecv(&buf[0], 1, MPI INT, prev, tag1, MPI COMM WORLD, &reqs[0]);
MPI Irecv(&buf[1], 1, MPI INT, next, tag2, MPI COMM WORLD, &reqs[1]);
MPI Isend(&rank, 1, MPI INT, prev, tag2, MPI COMM WORLD, &reqs[2]);
MPI Isend(&rank, 1, MPI INT, next, tag1, MPI COMM WORLD, &reqs[3]);
   { do some work that does not depend on the data being received }
MPI Waitall(4, reqs, stats);
                     Nearest neighbor exchange
MPI Finalize();
                     in a ring topology
```

Collective communication routines

- Use these when communicating among processes with a well defined pattern
- Some can be used to allow all processes to communicate
- Some perform computation during the communication (reductions)
- Involve all processes in the specified communicator, even if a particular processor has no data to send
- Can only be used with MPI predefined types, not derived types.
- The programmer has to make sure all processes participate in the collective operation

All processors participate in the collective operation

```
if (pid % 2) {
    MPI_Reduce(..., MPI_COMM_WORLD);
}
```

This program will deadlock, as the MPI_Reduce will wait forever for even processes to begin executing it.

If you want to only involve odd processes, add them to a new communicator.

Groups and communicators

- Two terms used in MPI documentation are groups and communicators.
- A communicator is a group of processes that can communicate with each other
- A *group* is an ordered set of processes
- Programmers can view groups and communicators as being identical

Collective routines

MPI_Barrier (comm): tasks block upon reaching the barrier until every task in the group has reached it

MPI_Bcast (&buffer,count,datatype,root,comm): process *root* sends a copy of its data to every other processor. Should be log2(comm_size) operation.

MPI_Scatter (&sendbuf,sendcnt,sendtype,&recvbuf, recvcnt,recvtype,root,comm): distributes a unique message from *root* to every process in the group.

Collective routines

MPI_Gather(&sendbuf, sendcnt, sendtype, &recvbuf, recvcount, recvtype, root, comm):

opposite of scatter, every process in the group sends a unique message to the *root*.

MPI_Allgather (&sendbuf,sendcount,sendtype,&recvbuf, recvcount,recvtype,comm): each tasks performs a one-to-all broadcast to every other process in the group These are concatenated together in the recvbuf.

MPI_Reduce (&sendbuf,&recvbuf,count,datatype,op,root,comm): performs a reduction using operation *op* and places the result into *recvbuf* on the *root* process.

MPI Bcast

MPI_Bcast

Broadcasts a message to all other processes of that group

MPI Scatter

Equivalent to MPI_Send(sendbuf+i*sendcount*extent(sendtype), sendcount, sendtype, i, ...) MPI Recv(recvbuf, recvcount, recvtype, i, sendcount, sendtype, i, ...)

MPI_Scatter

Sends data from one task to all other tasks in a group

```
sendcnt = 1;
recvent = 1;
                   task 1 contains the message to be scattered
src = 1;
MPI Scatter(sendbuf, sendcnt, MPI INT,
             recvbuf, recvent, MPI_INT,
             src, MPI COMM WORLD);
task 0
             task 1
                          task 2
                                       task 3
               1
               2
                                                       sendbuf (before)
               3
               4
  1
                                                       recybuf (after)
               2
                            3
                                         4
```

MPI_Gather

Equivalent to MPI_Send(sendbuf, sendcount, sendtype, root, ...) MPI_Recv(recvbuf+i*recvcount*extent(recvtype), recvcount, recvtype, i, ...) With the results of each recv stored in rank order of the sending process

MPI_Gather

Gathers together values from a group of processes

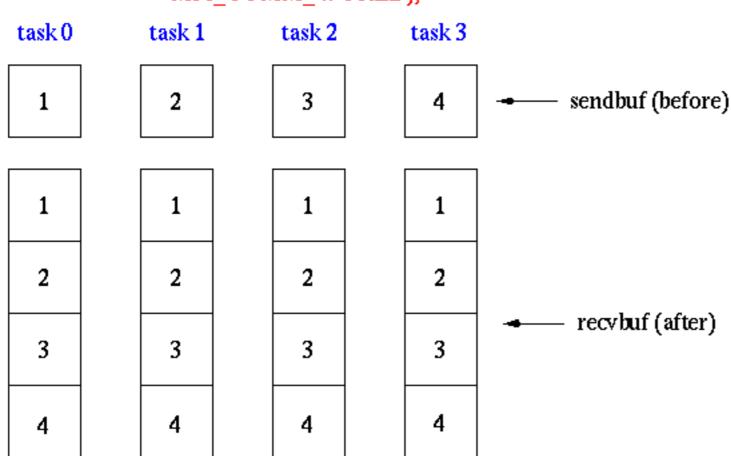
```
sendcnt = 1;
recvent = 1;
src = 1;
                   messages will be gathered in task 1
MPI_Gather(sendbuf, sendcnt, MPI_INT,
             recvbuf, recvent, MPI_INT,
             src, MPI COMM WORLD);
task 0
             task 1
                          task 2
                                       task 3
                                                       sendbuf (before)
                            3
  1
               2
                                         4
               1
               2
                                                       recybuf (after)
               3
               4
```

MPI_Allgather

MPI_Allgather

Gathers together values from a group of processes and distributes to all

A gather with every process being a target.



MPI_Reduce

recybuf (after)

MPI_Reduce

Perform and associate reduction operation across all tasks in the group and place the result in one task

10

```
count = 1;
dest = 1; result will be placed in task 1
MPI_Reduce(sendbuf, recvbuf, count, MPI_INT, MPI_SUM,
dest, MPI_COMM_WORLD);
task 0 task 1 task 2 task 3

1 2 3 4 sendbuf (before)
```

Also see MPI introductory slides.

You can form your own reduction function using MPI Op create

MPI_Op_create

```
#include "mpi.h" int MPI_Op_create(MPI_User_function *function, int commute, MPI_Op *op )
```

pointer to the user defined Function that is the *Op*

true if commutative, false otherwise

Handle to refer to the function wherever an MPI_Op is needed

More operations

MPI_Allreduce (&sendbuf, &recvbuf, count, datatype, op, comm):

functionally equivalent to an MPI_Reduce followed by an MPI_Bcast. Faster on most hardware than the combination of these.

MPI_Reduce_scatter(&sendbuf, &recvbuf, recvcount, datatype, op, comm):

Does an element-wise reduce on the vector in *sendbuf* of length *recvcount*. The vector is then split into disjoint segments and spread across the tasks. Equivalent to an *MPI_Reduce* followed by an *MPI_Scatter* operation.

More operations

MPI_Alltoall(&sendbuf, sendcount, sendtype, &recvbuf, recvcnt, recvtype, comm):

Each task in the group performs a scatter with the results concatenated on each process in task rank order.

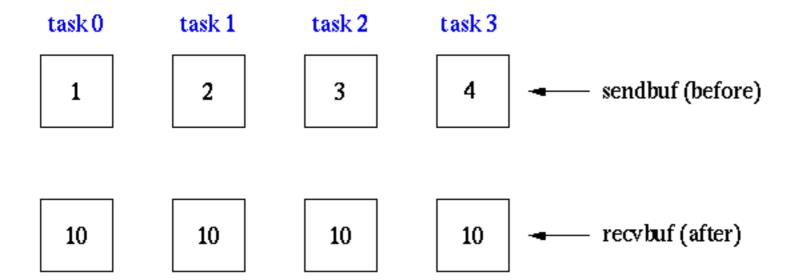
MPI_Scan(&sendbuf, &recvbuf, count, datatype, op, comm):

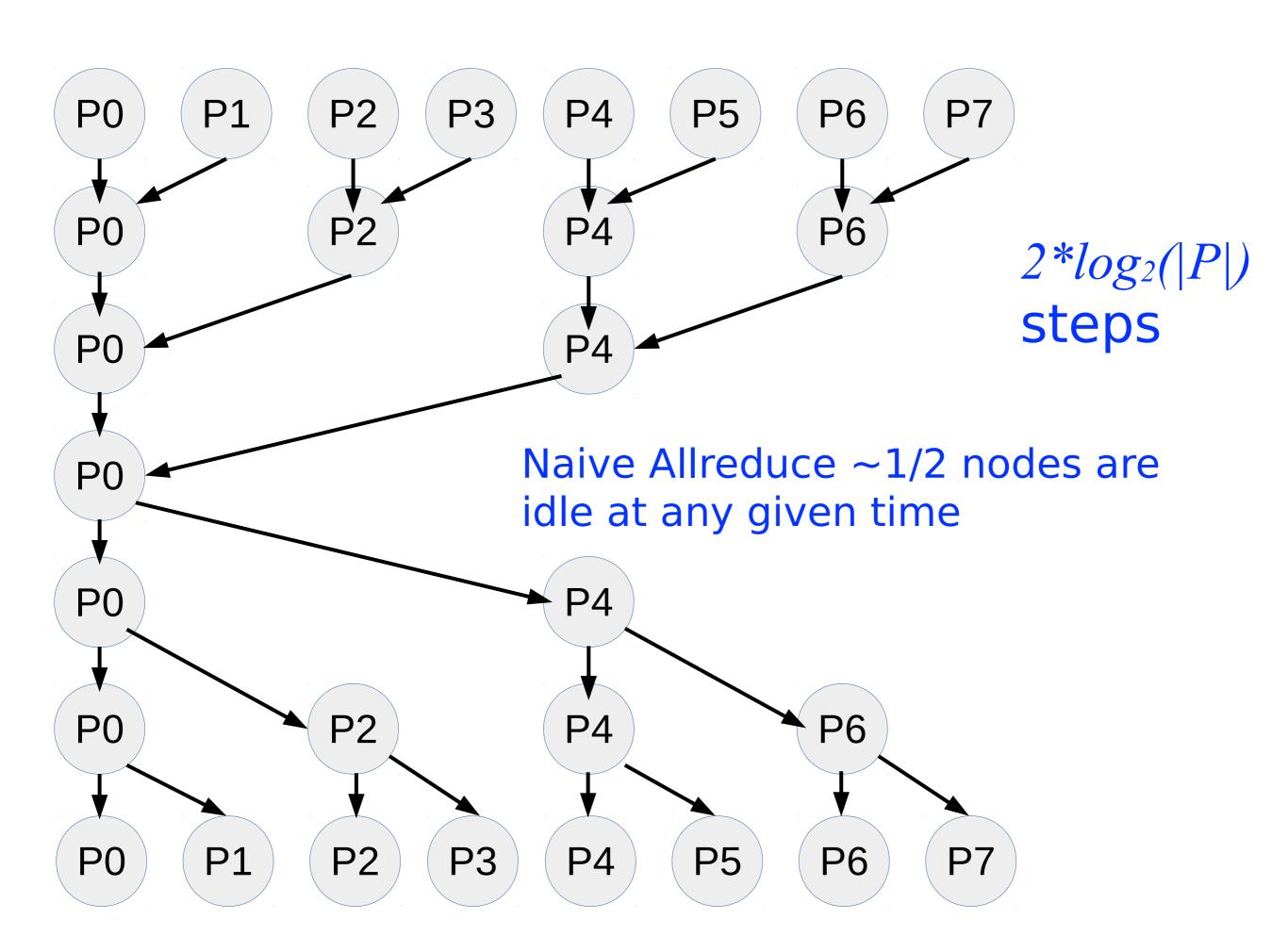
performs the partial sums on each processor that would result from doing an in-order reduction across the processors in rank order.

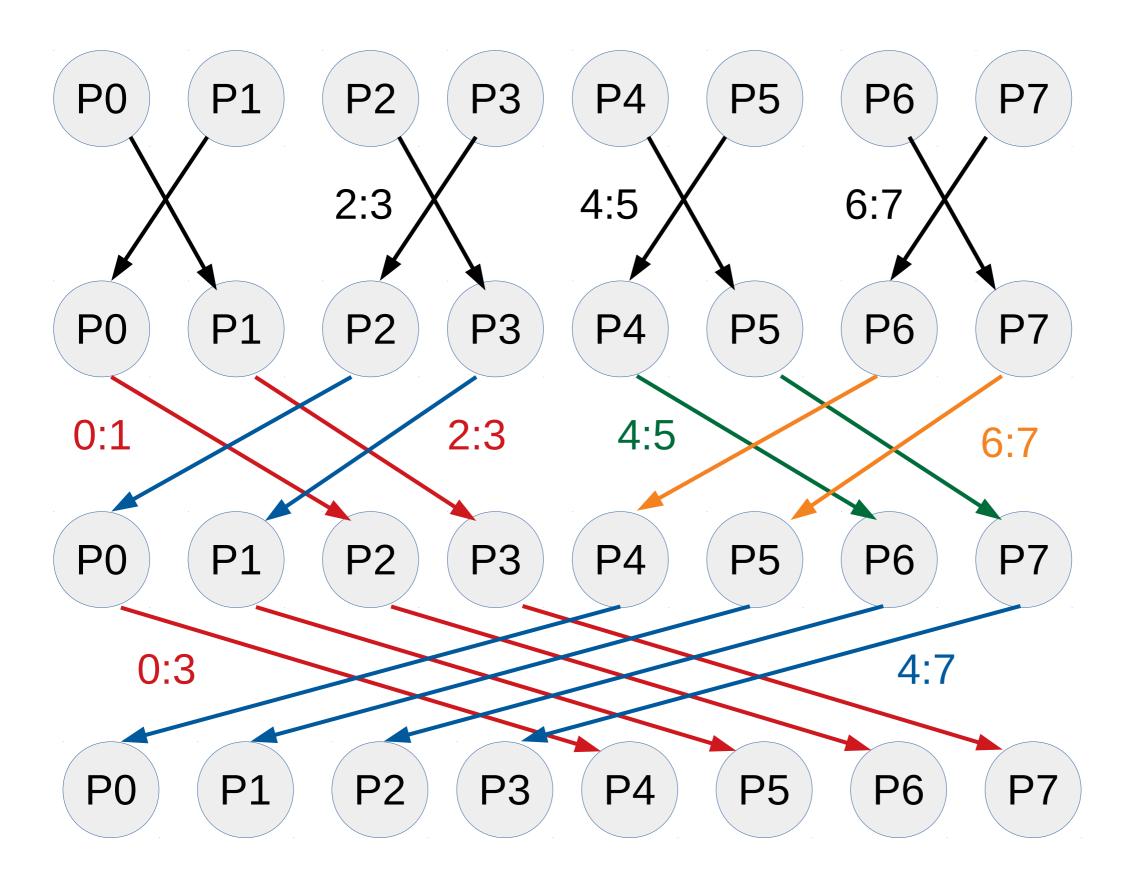
MPI_Allreduce

MPI Allreduce

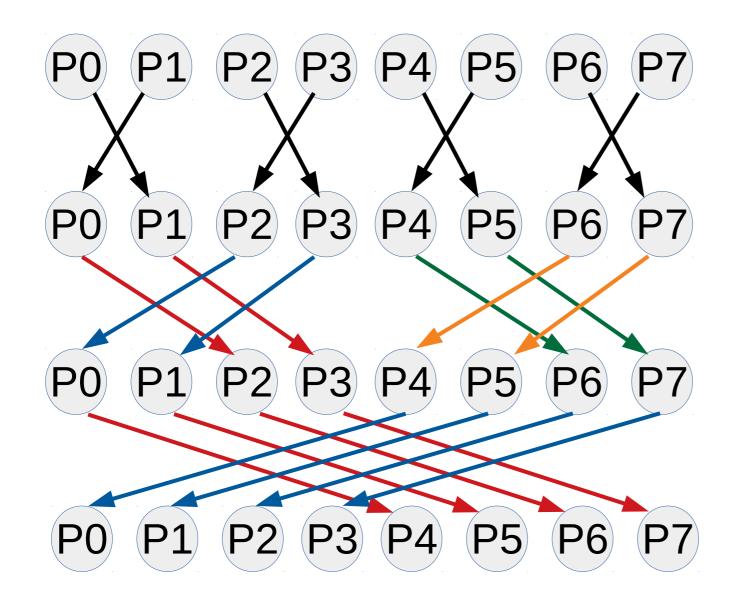
Perform and associate reduction operation across all tasks in the group and place the result in all tasks







 $log_2(|P|)$ steps



All processors all busy each step.

Note that the bandwidth requirements of the network change

Algorithm from *Optimization of Collective Reduction Operations*, Rolf Rabenseifner, International Conference on Computational Science, 2004

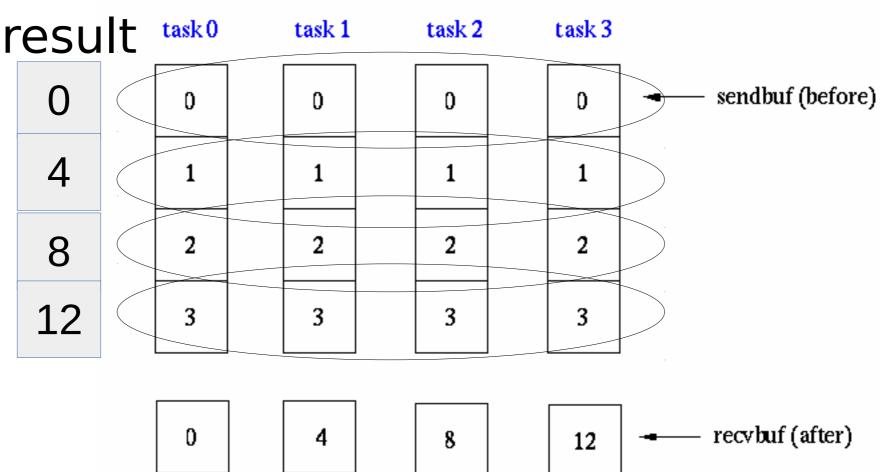
MPI_Reduce_scatter

MPI_Reduce_scatter

Perform reduction operation on vector elements across all tasks in the group, then distribute segments of result vector to tasks

recvcount = 1;
MPI_Reduce_scatter(sendbuf, recvbuf, recvcount, MPI_INT, MPI_SUM,
MPI_COMM_WORLD);

reduce



MPI Alltoall

Sends data from all to all processes. Each process performs a scatter operation.

```
sendcnt = 1;
recvent = 1:
MPI Alltoall(sendbuf, sendcnt, MPI_INT,
             recvbuf, recvcnt, MPI INT,
             MPI CÓMM WORLD);
task 0
             task 1
                         task 2
                                      task 3
                                        13
               5
  1
                            9
  2
               6
                           10
                                        14
  3
                           11
                                        15
  4
               8
                           12
                                        16
  1
               2
                                        4
  5
               6
                                         8
                                                      recybuf (after)
  9
              10
                           11
                                        12
 13
              14
                           15
                                        16
```

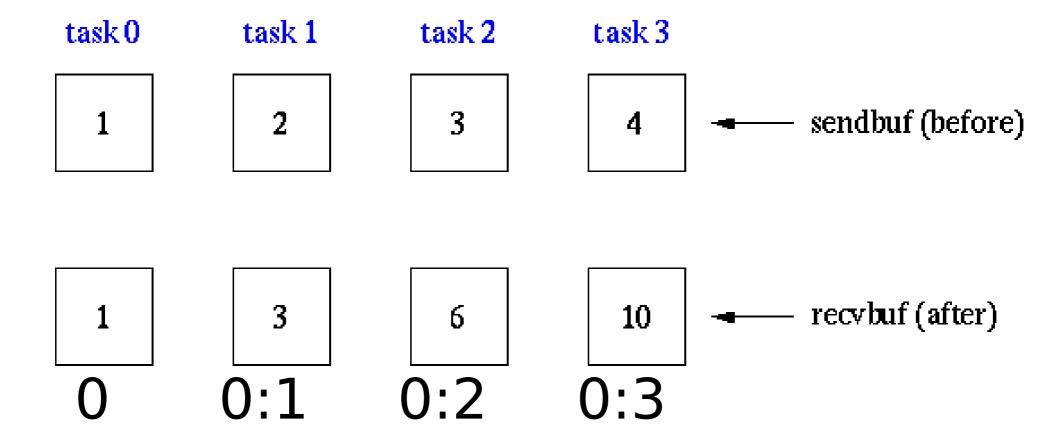
Each process performs a scatter of its elements to all other processes.

Received data is concatenated in sender rank order

sendbuf (before)

MPI_Scan

Computes the scan (partial reductions) of data on a collection of processes



Group and communicator

- Remember that
 - A communicator is a group of processes that can communicate with each other
 - A group is an ordered set of processes
- Programmers can view groups and communicators as being the same thing
- group routines are used in collecting processes to form communicator.

Why groups and communicators?

- Allow programmer to organize tasks by functions
- Enable collective communication operations
- Allow user-defined virtual topologies to be formed
- Enable manageable communication by enabling synchronization

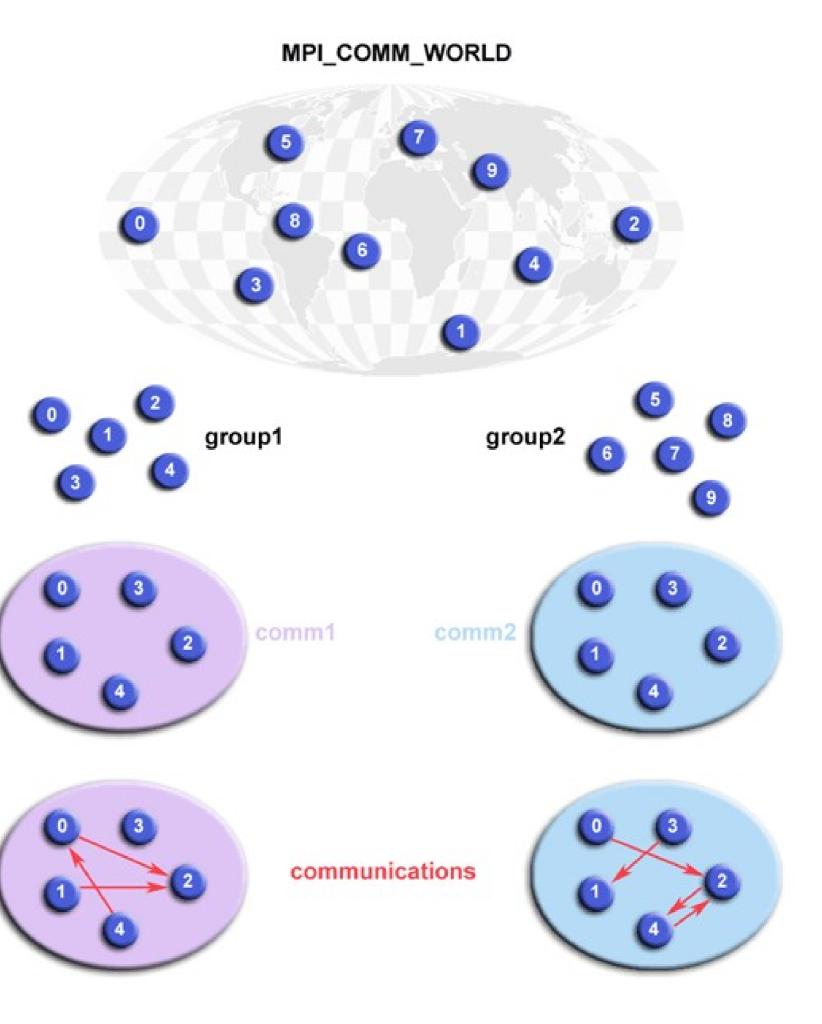
Properties

- Groups/communicators are dynamic, i.e. they can be created and destroyed
- Processes can be in many groups, and will have a unique, possibly different, rank in each group
- MPI provides 40+ routines for managing groups and communicators! Mercifully, we will not cover them all.

functions of these 40+ routines

- Extract handle of a global group and communicator using MPI_Comm_group
- Form new group as a subset of another group using MPI_Group_incl
- Create new communicator for a group using MPI_Comm_create
- Determine a processor's rank in a communicator using MPI_Comm_rank
- Communicate among the processors of a group
- When finished, free communicators and groups using MPI_Comm_free and MPI_Group_free

Relationships among communicators and groups.



Both collective and point-to-point communication is within a group.

```
Handle for
#include "mpi.h"
                       MPI COMM WORLD
#include <stdio.h>
                             group
#define NPROCS 8
                              Handle for a
                                                  Get the number
int main(argc,argv)
                               new group
                                                     of tasks and
int argc;
                                                     the rank of
char *argv[]; {
                                                 MPI COMM WORLD
     rank, new_rank, sendbuf, recvbuf, numtasks,
int
                                                   for this process
      ranks1[4]=\{0,1,2,3\}, ranks2[4]=\{4,5,6,7\};
MPI_Group orig_group, new_group;
Handle for a
MPI Comm new comm;
                                    new
                               communicator
MPI Init(&argc,&argv);
MPI Comm rank(MPI COMM WORLD, &rank);
MPI Comm size(MPI COMM WORLD, &numtasks);
if (numtasks != NPROCS) {
 printf("Must specify "AP PROCS= %d. Terminating.\n", NPROCS);
 MPI Finalize();
                                           sanity check
 exit(0);
                                               code
```

```
Group this will be in. Note that since this is an
#include "mpi.h"
                          SPMD program, if we do this statically we
#include <stdio.h>
#define NPROCS 8
                         need information for all groups the process
                            can be in, not just the one that it is in.
int main(argc,argv)
int argc;
char *argv[]; {
      rank, new rank, sendbuf, recvbuf, numtasks,
int
      ranks1[4]=\{0,1,2,3\}, ranks2[4]=\{4,5,6,7\};
MPI Group orig group, new soup;
MPI Comm new_comm;
                                   Hold the ranks of processors
                                     in (in MPI COMM WORLD)
MPI Init(&argc,&argv);
                                    of processes in each of the
MPI Comm rank(MPI COMM WORLD
                                          two new groups.
MPI_Comm_size(MPI_COMM WORLD,
if (numtasks != NPROCS) {
 printf("Must specify MP PROCS= %d. Terminating.\n", NPROCS);
 MPI Finalize();
```

exit(0);

Variables to hold information about the new

get handle for MPI COMM WORLD

```
sendbuf = rank;
/* Extract the original group handle */
MPI Comm group(MPI COMM WORLD, &orig group);
/* Divide tasks into two distinct groups based upon rank */
if (rank < NPROCS/2) {
 MPI_Group_incl(orig_group, NPROCS/2, ranks1, &new_group);
else {
 MPI_Group_incl(orig_group, NPROCS/2, ranks2, &new_group);
                           Each process executes one of
                                                                ations */
/* Create new new commun
                            the if branches. Based on
MPI_Comm_create(MPI_C(
                             its number, each process
MPI Allreduce(&sendbuf, &
                               becomes a member of
                              one of the new groups.
MPI_Group_rank (new_group_rank)
printf("rank= %d newrank=
MPI Finalize();
```

```
Create a communicator
sendbuf = rank;
                           From the group formed
                                     above
/* Extract the original group handle */
MPI Comm_group(MPI_COMM_WORLD, &orig_group),
/* Divide tasks into two distinct groups based upon rank Perform collective
if (rank < NPROCS/2) {
                                             communication within the
 MPI_Group_incl(orig_group, NPROCS/2, ranks1, & necommunicator comm
else {
 MPI Group incl(orig group, NPROCS/2, ranks2, &new group);
/* Create new new communicator and then perform collective communications */
MPI_Comm_create(MPI_COMM_WORLD, new_group, &new_comm);
MPI Allreduce(&sendbuf, &recvbuf, 1, MPI_INT, MPI_SUM, new_comm);
MPI_Group_rank (new_group, &new_rank);
printf("rank= %d newrank= %d recvouf= %d\n",rank,new rank,recvbuf);
MPI Finalize();
                             Get the processes rank
                               within the new group
```